Lecture #14 of 18(?)

Time-Dependence in Electrochemistry

Chapters 4 and 5
Q: What’s in this set of lectures?
A: B&F Chapters 4 & 5 main concepts:

- Section 4.4.2: Fick’s Second Law of Diffusion
- Section 5.1: Overview of step experiments
- Section 5.2: Potential step under diffusion controlled
- Sections 5.3 & 5.9: Ultramicroelectrodes
- Sections 5.7 – 5.8: Chronoamperometry/Chronocoulometry
Looking forward... Section 4.4.2 and Chapter 5

- Fick’s Second Law of Diffusion
- Linear Diffusion = time-dependent current (Cottrell Equation)
- Anson Plots for surface adsorbed species
- Radial Diffusion = time-independent current (at steady-state)
- Ultramicroelectrodes (UMEs)
- Scanning Electrochemical Microscopy (SECM)
- Single molecule electrochemistry
RECALL:
... so in a potential step experiment...

1. current changes continuously with time.

2. radial diffusion (AKA “edge effects”) limits the data acquisition time window to ~1 – 10 s.

3. charging imposes a lower limit of 0.1 – 0.5 µs on this data acquisition time window.

4. theoretically, maximum current densities are > 60 mA cm\(^{-2}\) initially, but just 100 µA cm\(^{-2}\) at S/N ≈ 10.

... but, why do we care?
Why do we care? One reason...

**RECALL:**

we need to push this up in order to perform meaningful measurements of the kinetics of rapid redox reactions.

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**Figure 3.4.1** Current-overpotential curves for the system $O + e \rightleftharpoons R$ with $\alpha = 0.5$, $T = 298$ K, $i_{l,c} = -i_{l,a} = i_l$ and $i_0/i_l = 0.2$. The dashed lines show the component currents $i_c$ and $i_a$. 

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**Why do we care? One reason...**

**RECALL:**

we need to push this up in order to perform meaningful measurements of the kinetics of rapid redox reactions.
... but deleterious edge effects also suggest an opportunity:

**RECALL:**

What if instead of avoiding radial diffusion, we exploit it?... Wait, what?

How is current affected, relative to the Cottrell prediction?

$t \approx 0.1 - 1 \text{ s}$

*linear diffusion*

$t \approx 1 - 100 \text{ s}$

*mixed diffusion*

$t > \sim 100 \text{ s}$

*nemi-spherical diffusion*

3 mm

insulator

Au tip on metal rod
Let’s design an experiment in which we intentionally operate in this radial diffusion limit the “entire” time!

RECALL:

... well we actually start in the linear regime, and then switch over quickly...

side view     top view

10 nm – 50 μm

insulator

Au, C, Pt on metal rod

... called “ultramicroelectrodes” or “UMEs”
... steady-state is “often” reached at each applied potential at a UME during a sweep

... Let’s quantify it... steady-state occurs when $\nu << \frac{RTD}{(nFR_0^2)}$

... $\nu \text{ (mV s}^{-1} \text{)} << 26 \text{ mV} \times (D/r_0^2)$... for a BASi UME with $r_0 = 5 \mu \text{m}$...

... $26 \text{ mV} \times \left( \frac{(0.5 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1})}{(0.5 \times 10^{-3} \text{ cm})^2} \right) = 26 \text{ mV} \times (20 \text{ s}^{-1})$

... $\nu << 0.5 \text{ V s}^{-1}$... Wow, you can still scan quite fast!
... and back by popular demand: “Steps to convert TX to IUPAC”...

(1) Change sign of currents, because B&F indicates that positive current is cathodic
... this is likely because electrons are the charge carrier of current
... but physicists disagree; also, ions can be + or – so no need to focus on e⁻

(2) Mirror image the plot through the origin so that the axes look like math axes
… and back by popular demand: “Steps to convert TX to IUPAC”...

Figure 1. Cyclic voltammogram of ferrocene (1.0 mM) in acetonitrile with 0.1 M tetra-n-butylammonium perchlorate at a gold microdisk electrode \( (r = 6.5 \ \mu m) \).

(a) 10 V s\(^{-1} \) scan rate. (b) 0.1 V s\(^{-1} \) scan rate
... akin to the semi-empirical model of mass transfer (Section 1.4.2), UMEs have no scan-rate dependence to $i_l$, which is unlike CVs at traditional disk electrodes...

... this is because we can attain steady-state, mass-transport-limited currents at UMEs...

... without stirring at all!

Figure 3. Cyclic voltammograms of 1 mM ferrocene in 0.1 M TBAPF$_6$/CH$_3$CN solutions obtained with (a) 10; (b) 25; (c) 50 µm-diameter Pt disks. The scan rate is 50 mV/s.
and the diffusion-limited current pre-factor depends on electrode geometry...

RECALL:

\[ i_l = "x"nFDC^*r_0 \]

... but not scan rate!

<table>
<thead>
<tr>
<th>electrode geometry</th>
<th>&quot;x&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>sphere</td>
<td>4π</td>
</tr>
<tr>
<td>hemisphere</td>
<td>2π</td>
</tr>
<tr>
<td>disk</td>
<td>4</td>
</tr>
<tr>
<td>ring</td>
<td>[ \frac{\pi^2(b + a)}{r_0 \ln \left[ \frac{16(b + a)}{(b - a)} \right]} ]</td>
</tr>
</tbody>
</table>

... disk and ring–disk electrodes are real things that we will cover later.
... how tiny (or large) is this diffusion-limited current (density)? Let’s assume $D = 0.5 \times 10^{-5}$ cm$^2$ s$^{-1}$ and $C^* = 10^{-6}$ moles cm$^{-3}$ (1 mM)

<table>
<thead>
<tr>
<th>$r_0$</th>
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<th>$i_l/A = i_l/(\pi r_0^2)$</th>
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<td>100 µm</td>
<td>20 nA</td>
<td>60 µA/cm$^2$</td>
</tr>
<tr>
<td>10 µm</td>
<td>2 nA</td>
<td>0.6 mA/cm$^2$</td>
</tr>
<tr>
<td>1 µm</td>
<td>0.2 nA</td>
<td>6 mA/cm$^2$</td>
</tr>
<tr>
<td>100 nm</td>
<td>20 pA</td>
<td>60 mA/cm$^2$</td>
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<td>0.2 pA</td>
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... super tiny currents...  ... but huge current densities... = AWESOME!...  ... for studies of electrocatalysis!
Again, Why do we care? One reason...

we need to push this up in order to perform meaningful measurements of the kinetics of rapid redox reactions.

Figure 3.4.1 Current-overpotential curves for the system \( O + e \rightleftharpoons R \) with \( \alpha = 0.5, \ T = 298 \) K, \( i_{l,c} = -i_{l,a} = i_l \) and \( i_0/i_l = 0.2 \). The dashed lines show the component currents \( i_c \) and \( i_a \).
... how tiny (or large) is this diffusion-limited current (density)?

Let’s assume \( D = 0.5 \times 10^{-5} \) cm\(^2\) s\(^{-1}\) and \( C^* = 10^{-6} \) moles cm\(^{-3}\) (1 mM)

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<td>0.2 nA</td>
<td>6 mA/cm(^2)</td>
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... so what is \( iR_u \) for these electrodes?

... well \( R_u \) is fairly large, but \( iR_u \) is nearly constant...

\[
R_u = \frac{1}{4\pi kr_0} \left( \frac{x}{x + r_0} \right)
\]
Recall this? This was measured using a UME! Reaction is $\text{Tl}^+ + e^- \rightleftharpoons \text{Tl}^0$

**Figure 4.3.5** Voltammograms for reduction of 0.65 mM Tl$_2$SO$_4$ at a mercury film on a silver ultramicroelectrode (radius, 15 $\mu$m) in the presence of (a) 0, (b) 0.1, (c) 1, and (d) 100 mM LiClO$_4$. The potential was controlled vs. a Pt wire QRE whose potential was a function of solution composition. This variability is the basis for the shifts in wave position along the potential axis. [Reprinted with permission from M. Ciszkowska and J. G. Osteryoung, *Anal. Chem.*, 67, 1125 (1995). Copyright 1995, American Chemical Society.]
Fig. 2. Voltammogram for oxidation of $10^{-3}$ M ferrocene in acetonitrile at 18 °C using a 25-μm radius Pt microelectrode in (a) presence and (b) absence of 0.1 M $\text{Et}_4\text{NClO}_4$. Scan rate = 5 mV s$^{-1}$. 

... additional/final points to address about UMEs:

1) You can buy them; how do you make them?
2) UME arrays and ensembles
3) Potential step experiments with UMEs...
4) How rapidly is steady-state attained?
... arrays of UMEs (from B&F)...

Semi-infinite **linear** diffusion

Semi-infinite **mixed** diffusion

**Figure 5.2.4** Evolution of the diffusion field during chronoamperometry at an electrode with active and inactive areas on its surface. In this case the electrode is a regular array such that the active areas are of equal size and spacing, but the same principles apply for irregular arrays. (a) Short electrolysis times, (b) intermediate times, (c) long times. Arrows indicate flux lines to the electrode.

Semi-infinite linear diffusion, *again*

... no matter what some papers suggest... you can **never** beat overall linear diffusive flux over the total projected area of the diffusive solution region!
... this is because a UME must behave like a macroscopic electrode at sufficiently small times (and if neighboring electrodes are too close), right?... *Right!*

macroscopic planar electrode:  \[ i(t) = \frac{nFAD^{1/2}C^*}{\pi^{1/2}t^{1/2}} \]

disk UME:  \[ i(t) = \frac{nFAD^{1/2}C^*}{\pi^{1/2}t^{1/2}} + 4nFDC^*r_0 \]

hemispherical UME:  \[ i(t) = \frac{nFAD^{1/2}C^*}{\pi^{1/2}t^{1/2}} + 2\pi nFDC^*r_0 \]
... additional/final points to address about UMEs:

1) You can buy them; how do you make them?
2) UME arrays and ensembles... **GOT IT!**
3) Potential step experiments with UMEs...
4) How rapidly is steady-state attained?
... at longer times, the current asymptotically approaches $i_f$. ...
... at longer times, the current asymptotically approaches $i_f$. 

- $r_0 = 10 \, \mu m$
- $C^* = 1 \times 10^{-6} \, \text{moles cm}^{-3}$
- $D = 0.5 \times 10^{-5} \, \text{cm}^2 \, \text{s}^{-1}$
note that the \( i(t) \) versus \((1/t^{1/2})\) plot no longer intersects “0”...

... and \( D \) can be calculated without knowing \( C^* \)...

How?

\[
i(t) = \frac{nFAD^{1/2}C^*}{\pi^{1/2}t^{1/2}} + 4nFDC^*r_0
\]

This is two equations!...

... calculate \( D^{1/2}C^* \) from slope...

and \( DC^* \) from y-intercept
... how long does it take for UME’s to attain steady-state?

\[ \text{Error} = \frac{nFAD^{1/2}C^*}{\pi^{1/2}t^{1/2}} + 4nFDC^*r_0 \]

For different values of \( r_0 \):
- \( r_0 = 10 \, \mu m \)
- \( 5 \, \mu m \)
- \( 2 \, \mu m \)
- \( 1 \, \mu m \)

Graph showing error over time for different values of \( r_0 \).
Recall that these diffusional processes only dominate in the absence of stirring at the electrode surface.

\[ r_0 = 10 \mu m \]

... how long does it take for UME’s to attain steady-state?
the short time limit imposed by $R_u C_d$ for a macroscopic electrode is ~100 µs...

... but UMEs charge much faster!

<table>
<thead>
<tr>
<th>$r_0$</th>
<th>$t_{\text{steady-state}}$</th>
<th>$R_u C_d^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-3}$ cm</td>
<td>25 s</td>
<td>1.7 µs</td>
</tr>
<tr>
<td>$10^{-4}$ cm</td>
<td>0.25 s</td>
<td>170 ns</td>
</tr>
<tr>
<td>$10^{-5}$ cm</td>
<td>2.5 ms</td>
<td>17 ns</td>
</tr>
<tr>
<td>$10^{-6}$ cm</td>
<td>25 µs</td>
<td>1.7 ns</td>
</tr>
</tbody>
</table>

... recall that $R_u$ is nearly independent of the WE–RE separation:

$$R_u = \frac{1}{4\pi kr_0} \left( \frac{x}{x + r_0} \right)$$
needed for $i_i < 10 \text{ nA}$: a Faraday cage...  

... and a Keithley 428 programmable current amplifier grounded to the cage
... there are also a few things you will *not* be needing...

- ground the Faraday cage to the Keithley ground
  - a reference electrode (no counter electrode needed)
  - Keithley 428
  - UME
UME take-home messages:

After rapid double-layer charging...

... operate UMEs at either short times
   = linear diffusion...
       Can determine effects of rapid catalysis without complications from double-layer charging

... or long times
   = steady-state radial diffusion...
       Can determine $D$ without knowing $C^*$
We only have time to look at techniques/publications by one of these four UME pioneers... **Guess who?**

- Ralph “Buzz” Adams (d. Univ. Kansas)
- Mark Wightman (UNC Chapel Hill)
- Allen Bard (UT Austin)
- Henry White (Univ. Utah)
Bard and scanning electrochemical microscopy (SECM)...
Fig. 5. Substrate imaging. (A) Tip is maintained at constant height while scanning horizontally. (B) SECM image of a portion (1 $\mu$m x 1 $\mu$m) of a human breast cell membrane acquired with a 47 nm radius ultramicroelectrode.
Fig. 7. (A) 80 μm × 80 μm atomic force microscopy image of a disk-like region of boron-doped diamond. (B) SECM image over such a region.

http://electrochem.cwru.edu/ed/encycl/art-m04-microscopy.htm
Electrochemical Detection of Single Molecules

Fu-Ren F. Fan and Allen J. Bard*

The electrochemical behavior of a single molecule can be observed by trapping a small volume of a dilute solution of the electroactive species between an ultramicroelectrode tip with a diameter of ~15 nanometers and a conductive substrate. A scanning electrochemical microscope was used to adjust the tip-substrate distance (~10 nanometers), and the oxidation of [(trimethylammonio)methyl] ferrocene (Cp₂FeTMA⁺) to Cp₂FeTMA²⁺ was carried out. The response was stochastic, and anodic current peaks were observed as the molecule moved into and out of the electrode-substrate gap. Similar experiments were performed with a solution containing two redox species, ferrocene carboxylate (Cp₂FeCOO⁻) and Os(bpy)₃²⁺ (bpy is 2,2'-bipyridyl).

Fig. 1. Idealized schematic illustration of the tip geometry and the tip-substrate configuration used.
the magic of “thin layer electrochemistry”...

\[ J_i(x) = -D_i \frac{C_0^* - C_0(x = 0)}{\partial_o} \]

... Recalling Section 1.4.2 (Semi-Empirical)
A feasibility assessment... 1 molecule is trapped within a 1 µm x 1 µm x 10 nm volume between an SECM tip and a surface. What’s the value of the limiting current?

\[
i = -n F A D \frac{\Delta C}{\delta}
\]

\[
C_{1\text{ molecule}} = (1 \text{ molecule}) \left( \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ molecules}} \right) \left( \frac{1}{(10 \times 10^{-7} \text{ cm})(1 \times 10^{-4} \text{ cm})^2} \right)
\]

\[
C_{1\text{ molecule}} = 1.66 \times 10^{-10} \text{ mol/cm}^3
\]
A feasibility assessment... 1 molecule is trapped within a 1 µm x 1 µm x 10 nm volume between an SECM tip and a surface. What’s the value of the limiting current?

\[ i = -nFAD \frac{\Delta C}{\delta} \]

\[ i = (1 \text{ eq/mol})(96485 \text{ C/eq})(1 \times 10^{-4} \text{ cm})^2(1 \times 10^{-5} \text{ cm}^2/\text{s}) \left( \frac{1.66 \times 10^{-10} \text{ mol/cm}^3}{10 \times 10^{-7} \text{ cm}} \right) \]

\[ = 1.6 \times 10^{-12} \text{ A} = 1.6 \text{ pA} \]

... so we’re talking about pA's. We can measure that!
Fig. 2. Dependence of tip current on relative tip displacement over a conductive ITO substrate in a solution containing 2 mM Cp₂FeTMA⁺ and 2.0 M NaNO₃. The ITO substrate was biased at −0.3 V versus SCE and the tip was biased at 0.55 V, where the redox reactions on both electrodes were diffusion-controlled. The tip moved to the substrate surface at a rate of 30 Å/s. The data were 300 points averaged at each location. The significance of points A, B, and E is discussed in the text. See also Fig. 4.

Fan & Bard, Science, 1995, 267, 871

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- Sections 5.3 & 5.9: Ultramicroelectrodes
- Sections 5.7 – 5.8: Chronoamperometry/Chronocoulometry
A detailed review of Section 4.4.2 and Chapter 5

- Fick’s Second Law of Diffusion
- Linear Diffusion = time-dependent current (Cottrell Equation)
- Anson Plots for surface adsorbed species
- Radial Diffusion = time-independent current (at steady-state)
- Ultramicroelectrodes (UMEs)
- Scanning Electrochemical Microscopy (SECM)
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