## CHAPTER 4: SYMMETRY AND GROUP THEORY

4.1 a. Ethane in the staggered conformation has $2 C_{3}$ axes (the $\mathrm{C}-\mathrm{C}$ line), 3 perpendicular $C_{2}$ axes bisecting the $\mathrm{C}-\mathrm{C}$ line, in the plane of the two C's and the H's on opposite sides of the two C's. No $\sigma_{h}, 3 \sigma_{d}, i, S_{6} . \boldsymbol{D}_{3 d}$.
b. Ethane in eclipsed conformation has two $C_{3}$ axes (the $\mathrm{C}-\mathrm{C}$ line), three perpendicular $C_{2}$ axes bisecting the $\mathrm{C}-\mathrm{C}$ line, in the plane of the two C 's and the H's on the same side of the two C's. Mirror planes include $\sigma_{h}$ and $3 \sigma_{d .}$. $\boldsymbol{D}_{3 h}$.
c. Chloroethane in the staggered conformation has only one mirror plane, through both C's, the Cl , and the opposite H on the other C. $\boldsymbol{C}_{\boldsymbol{s}}$.
d. 1,2-dichloroethane in the trans conformation has a $C_{2}$ axis perpendicular to the $\mathrm{C}-\mathrm{C}$ bond and perpendicular to the plane of both Cl 's and both C 's, a $\sigma_{h}$ plane through both Cl's and both C's, and an inversion center. $\boldsymbol{C}_{2 h}$.
4.2 a. Ethylene is a planar molecule, with $C_{2}$ axes through the C 's and perpendicular to the $\mathrm{C}-\mathrm{C}$ bond both in the plane of the molecule and perpendicular to it. It also has a $\sigma_{h}$ plane and two $\sigma_{d}$ planes (arbitrarily assigned). $\boldsymbol{D}_{2 h}$.
b. Chloroethylene is also a planar molecule, with the only symmetry element the mirror plane of the molecule. $\boldsymbol{C}_{\boldsymbol{s}}$.
c. 1,1-dichloroethylene has a $C_{2}$ axis coincident with the $\mathrm{C}-\mathrm{C}$ bond, and two mirror planes, one the plane of the molecule and one perpendicular to the plane of the molecule through both C's. $\boldsymbol{C}_{2}$.
cis-1,2-dichloroethylene has a $C_{2}$ axis perpendicular to the $\mathrm{C}-\mathrm{C}$ bond, and in the plane of the molecule, two mirror planes (one the plane of the molecule and one perpendicular to the plane of the molecule and perpendicular to the $\mathrm{C}-\mathrm{C}$ bond). $\boldsymbol{C}_{2}$.
trans-1,2-dichloroethylene has a $C_{2}$ axis perpendicular to the $\mathrm{C}-\mathrm{C}$ bond and perpendicular to the plane of the molecule, a mirror plane in the plane of the molecule, and an inversion center. $\boldsymbol{C}_{2 h}$.
4.3 a. Acetylene has a $C_{\infty}$ axis through all four atoms, an infinite number of perpendicular $C_{2}$ axes, a $\sigma_{h}$ plane, and an infinite number of $\sigma_{d}$ planes through all four atoms. $\boldsymbol{D}_{\infty h}$.
b. Fluoroacetylene has only the $C_{\infty}$ axis through all four atoms and an infinite number of mirror planes, also through all four atoms. $\boldsymbol{C}_{\infty v}$.
c. Methylacetylene has a $C_{3}$ axis through the carbons and three $\sigma_{v}$ planes, each including one hydrogen and all three C's. $\boldsymbol{C}_{3 v}$.
d. 3-Chloropropene (assuming a rigid molecule, no rotation around the $\mathrm{C}-\mathrm{C}$ bond) has no rotation axes and only one mirror plane through Cl and all three C atoms. $\boldsymbol{C}_{\boldsymbol{s}}$.
e. Phenylacetylene (again assuming no internal rotation) has a $C_{2}$ axis down the long axis of the molecule and two mirror planes, one the plane of the benzene ring and the other perpendicular to it. $\boldsymbol{C}_{2 v}$
4.4 a. Napthalene has three perpendicular $C_{2}$ axes, and a horizontal mirror plane (regardless of which $C_{2}$ is taken as the principal axis), making it a $\boldsymbol{D}_{2 h}$ molecule.
b. 1,8-dichloronaphthalene has only one $C_{2}$ axis, the $\mathrm{C}-\mathrm{C}$ bond joining the two rings, and two mirror planes, making it a $\boldsymbol{C}_{2 v}$ molecule.
c. 1,5-dichloronaphthalene has one $C_{2}$ axis perpendicular to the plane of the molecule, a horizontal mirror plane, and an inversion center; overall, $\boldsymbol{C}_{2 \boldsymbol{h}}$.
d. 1,2-dichloronaphthalene has only the mirror plane of the molecule, and is a $\boldsymbol{C}_{\boldsymbol{s}}$ molecule.
4.5 a. 1, $1^{\prime}$-dichloroferrocene has a $C_{2}$ axis parallel to the rings, perpendicular to the $\mathrm{Cl}-\mathrm{Fe}-\mathrm{Cl}$ $\sigma_{h}$ mirror plane. It also has an inversion center. $\boldsymbol{C}_{2 h}$.
b. Dibenzenechromium has collinear $C_{6}, C_{3}$, and $C_{2}$ axes perpendicular to the rings, six perpendicular $C_{2}$ axes and a $\sigma_{h}$ plane, making it a $\boldsymbol{D}_{6 h}$ molecule. It also has three $\sigma_{v}$ and three $\sigma_{d}$ planes, $S_{3}$ and $S_{6}$ axes, and an inversion center.
c. Benzenebiphenylchromium has a mirror plane through the Cr and the biphenyl bridge bond and no other symmetry elements, so it is a $\boldsymbol{C}_{\boldsymbol{s}}$ molecule.
d. $\quad \mathrm{H}_{3} \mathrm{O}^{+}$has the same symmetry as $\mathrm{NH}_{3}:$ a $C_{3}$ axis, and three $\sigma_{v}$ planes for a $\boldsymbol{C}_{3 v}$ molecule.
e. $\quad \mathrm{O}_{2} \mathrm{~F}_{2}$ has a $C_{2}$ axis perpendicular to the $\mathrm{O}-\mathrm{O}$ bond and perpendicular to a line connecting the fluorines. With no other symmetry elements, it is a $\boldsymbol{C}_{2}$ molecule.
f. Formaldehyde has a $C_{2}$ axis collinear with the $\mathrm{C}=\mathrm{O}$ bond, a mirror plane including all the atoms, and another perpendicular to the first and including the C and O atoms. $\boldsymbol{C}_{\mathbf{2 v}}$.
g. $\quad \mathrm{S}_{8}$ has $C_{4}$ and $C_{2}$ axes perpendicular to the average plane of the ring, four $C_{2}$ axes through opposite bonds, and four mirror planes perpendicular to the ring, each including two S atoms. $\boldsymbol{D}_{4 d}$.
h. Borazine has a $C_{3}$ axis perpendicular to the plane of the ring, three perpendicular $C_{2}$ axes, and a horizontal mirror plane. $\boldsymbol{D}_{3 h}$.
i. Tris(oxalato)chromate(III) has a $C_{3}$ axis and three perpendicular $C_{2}$ axes, each splitting a $\mathrm{C}-\mathrm{C}$ bond and passing through the $\mathrm{Cr} . \boldsymbol{D}_{3}$.
j. A tennis ball has three perpendicular $C_{2}$ axes (one through the narrow portions of each segment, the others through the seams) and two mirror planes including the first rotation axis. $\boldsymbol{D}_{2 d}$.
4.6 a. Cyclohexane in the chair conformation has a $C_{3}$ axis perpendicular to the average plane of the ring, three perpendicular $C_{2}$ axes between the carbons, and three $\sigma_{v}$ planes, each including the $C_{3}$ axis and one of the $C_{2}$ axes. $\boldsymbol{D}_{3 d}$.
b. Tetrachloroallene has three perpendicular $C_{2}$ axes, one collinear with the double bonds and the other two at $45^{\circ}$ to the $\mathrm{Cl}-\mathrm{C}-\mathrm{Cl}$ planes. It also has two $\sigma_{v}$ planes, one defined by each of the Cl-C-Cl groups. Overall, $\boldsymbol{D}_{2 d}$. (Note that the ends of tetrachlorallene are staggered.)
c. The sulfate ion is tetrahedral. $\boldsymbol{T}_{\boldsymbol{d}}$.
d. Most snowflakes have hexagonal symmetry (Figure 4.2), and have collinear $C_{6}, C_{3}$, and $C_{2}$ axes, six perpendicular $C_{2}$ axes, and a horizontal mirror plane. Overall, $\boldsymbol{D}_{6 h}$. (For high quality images of snowflakes, including some that have different shapes, see K. G. Libbrecht, Snowflakes, Voyageur Press, Minneapolis, MN, 2008.)
e. Diborane has three perpendicular $C_{2}$ axes and three perpendicular mirror planes. $\boldsymbol{D}_{2 h}$.
f. 1,3,5-tribromobenzene has a $C_{3}$ axis perpendicular to the plane of the ring, three perpendicular $C_{2}$ axes, and a horizontal mirror plane. $\boldsymbol{D}_{3 h}$.

1,2,3-tribromobenzene has a $\mathrm{C}_{2}$ axis through the middle Br and two perpendicular mirror planes that include this axis. $\boldsymbol{C}_{2 v}$

1,2,4-tribromobenzene has only the plane of the ring as a mirror plane. $\boldsymbol{C}_{\boldsymbol{s}}$.
g. A tetrahedron inscribed in a cube has $\boldsymbol{T}_{\boldsymbol{d}}$ symmetry (see Figure 4.6).
h. The left and right ends of $\mathrm{B}_{3} \mathrm{H}_{8}$ are staggered with respect to each other. There is a $C_{2}$ axis through the borons. In addition, there are two planes of symmetry, each containing four H atoms, and two $C_{2}$ axes between these planes and perpendicular to the original $C_{2}$. The point group is $\boldsymbol{D}_{2 d}$.
i. A mountain swallowtail butterfly has only a mirror that cuts through the head, thorax, and abdomen. $\boldsymbol{C}_{\boldsymbol{s}}$
j. The Golden Gate Bridge has a $\mathrm{C}_{2}$ axis and two perpendicular mirror planes that include this axis. $\boldsymbol{C}_{2 v}$
4.7 a. A sheet of typing paper has three perpendicular $C_{2}$ axes and three perpendicular mirror planes. $\boldsymbol{D}_{2 h}$.
b. An Erlenmeyer flask has an infinite-fold rotation axis and an infinite number of $\sigma_{v}$ planes, $\boldsymbol{C}_{\infty}$.
c. A screw has no symmetry operations other than the identity, for a $\boldsymbol{C}_{\mathbf{1}}$ classification.
d. The number 96 (with the correct type font) has a $C_{2}$ axis perpendicular to the plane of the paper, making it $\boldsymbol{C}_{2 h}$.
e. Your choice - the list is too long to attempt to answer it here.
f. A pair of eyeglasses has only a vertical mirror plane. $\boldsymbol{C}_{\boldsymbol{s}}$.
g. A five-pointed star has a $C_{5}$ axis, five perpendicular $C_{2}$ axes, one horizontal and five vertical mirror planes. $\boldsymbol{D}_{5 h}$.
h. A fork has only a mirror plane. $\boldsymbol{C}_{\boldsymbol{s}}$.
i. Wilkins Micawber has no symmetry operation other than the identity. $\boldsymbol{C}_{\mathbf{1}}$.
j. A metal washer has a $C_{\infty}$ axis, an infinite number of perpendicular $C_{2}$ axes, an infinite number of $\sigma_{v}$ mirror planes, and a horizontal mirror plane. $\boldsymbol{D}_{\infty h}$.
$4.8 \quad$ a. $\quad D_{2 h}$
b. $\quad D_{4 h}$ (note the four knobs)
c. $\quad C_{s}$
h. $\quad C_{8 v}$
d. $\quad C_{2 v}$
i. $\quad D_{\infty}$
e. $\quad C_{6 v}$
j. $\quad C_{3 v}$
4.9
a. $\quad D_{3 h}$
b. $\quad D_{4 h}$
c. $\quad C_{s}$
d. $\quad C_{3}$
e. $\quad C_{2 v}$
f. $\quad C_{s}$ (note holes)
g. $\quad C_{1}$
h. $\quad C_{3 v}$
i. $\quad D_{\infty h}$
j. $\quad C_{1}$
4.10 Hands (of identical twins): $C_{2} \quad$ Baseball: $D_{2 d} \quad$ Atomium: $C_{3 v}$

Eiffel Tower: $C_{4 v} \quad$ Dominoes: $6 \times 6: C_{2 v} \quad 3 \times 3: C_{2} \quad 5 \times 4: C_{s}$
Bicycle wheel: The wheel shown has 32 spokes. The point group assignment depends on how the pairs of spokes (attached to both the front and back of the hub) connect with the rim. If the pairs alternate with respect to their side of attachment, the point group is $D_{8 d}$. Other arrangements are possible, and different ways in which the spokes cross can affect the point group assignment; observing an actual bicycle wheel is recommended. (If the crooked valve is included, there is no symmetry, and the point group is a much less interesting $C_{1}$.)
4.11
a. Problem 3.41*:
a. $\mathrm{VOCl}_{3}: C_{3 v}$
b. $\mathrm{PCl}_{3}: C_{3 v}$
c. $\mathrm{SOF}_{4}: C_{2 v}$
d. $\mathrm{SO}_{3}: D_{3 h}$
e. $\mathrm{ICl}_{3}: C_{2 v}$
f. $\mathrm{SF}_{6}: O_{h}$
g. $\mathrm{IF}_{7}: D_{5 h}$
h. $\mathrm{XeO}_{2} \mathrm{~F}_{4}: D_{4 h}$
i. $\mathrm{CF}_{2} \mathrm{Cl}_{2}: C_{2 v}$
j. $\mathrm{P}_{4} \mathrm{O}_{6}: T_{d}$

[^0]b. Problem $3.42^{*}$ :
a. $\mathrm{PH}_{3}: C_{3 v}$
b. $\mathrm{H}_{2} \mathrm{Se}: C_{2 v}$
c. $\mathrm{SeF}_{4}: C_{2 v}$
d. $\mathrm{PF}_{5}: D_{3 h}$
e. $\mathrm{IF}_{5}: C_{4 v}$
f. $\mathrm{XeO}_{3}: C_{3 v}$
g. $\mathrm{BF}_{2} \mathrm{Cl}: C_{2 v}$
h. $\mathrm{SnCl}_{2}: C_{2 v}$
i. $\mathrm{KrF}_{2}: D_{\infty h}$
j. $\mathrm{IO}_{2} \mathrm{~F}_{5}{ }^{2-}: D_{5 h}$
a.

Figure 3.8:
a. $\mathrm{CO}_{2}: D_{\infty h}$
b. $\mathrm{SO}_{3}: D_{3 h}$
c. $\mathrm{CH}_{4}: T_{d}$
d. $\mathrm{PCl}_{5}: D_{3 h}$
e. $\mathrm{SF}_{6}: O_{h}$
f. $\mathrm{IF}_{7}: D_{5 h}$
g. $\mathrm{TaF}_{8}{ }^{3-}: D_{4 d}$
b.

Figure 3.15 :
a. $\mathrm{CO}_{2}: D_{\infty h}$
b. $\mathrm{COF}_{2}: C_{2 v}$
c. $\mathrm{NO}_{2}^{-}: C_{2 v}$
d. $\mathrm{SO}_{3}: \quad D_{3 h}$
e. $\mathrm{SNF}_{3}: C_{3 v}$
f. $\mathrm{SO}_{2} \mathrm{Cl}_{2}: C_{2 v}$
g. $\mathrm{XeO}_{3}: C_{3 v}$
h. $\mathrm{SO}_{4}{ }^{2-}: T_{d}$
i. $\mathrm{SOF}_{4}: C_{2 v}$
j. $\mathrm{ClO}_{2} \mathrm{~F}_{3}: C_{2 v}$
k. $\mathrm{XeO}_{3} \mathrm{~F}_{2}: D_{3 h}$

1. $\mathrm{IOF}_{5}: C_{4 v}$
4.13 a. $\quad p_{x}$ has $C_{\infty_{\nu}}$ symmetry. (Ignoring the difference in sign between the two lobes, the point group would be $D_{\infty h}$.)
b. $\quad d_{x y}$ has $D_{2 h}$ symmetry. (Ignoring the signs, the point group would be $D_{4 h}$.)
c. $\quad d_{x^{2}-y^{2}}$ has $D_{2 h}$ symmetry. (Ignoring the signs, the point group would be $D_{4 h}$.)
d. $\quad d_{z^{2}}$ has $D_{\infty h}$ symmetry.
e. $\quad f_{x y z}$ has $T_{d}$ symmetry.
4.14 a. The superimposed octahedron and cube show the matching symmetry elements.
The descriptions below are for the elements of a cube; each element also applies to the octahedron.
$E \quad$ Every object has an identity operation.
$8 C_{3}$ Diagonals through opposite corners of the cube are $C_{3}$ axes.

$6 C_{2} \quad$ Lines bisecting opposite edges are $C_{2}$ axes.
$6 C_{4} \quad$ Lines through the centers of opposite faces are $C_{4}$ axes. Although there are only three such lines, there are six axes, counting the $C_{4}{ }^{3}$ operations.
$3 C_{2} \quad\left(=C_{4}{ }^{2}\right)$ The lines through the centers of opposite faces are $C_{4}$ axes as well as $C_{2}$ axes.

[^1]$i \quad$ The center of the cube is the inversion center.
$6 S_{4} \quad$ The $C_{4}$ axes are also $S_{4}$ axes.
$8 S_{6} \quad$ The $C_{3}$ axes are also $S_{6}$ axes.
$3 \sigma_{h} \quad$ These mirror planes are parallel to the faces of the cube.
$6 \sigma_{d} \quad$ These mirror planes are through two opposite edges.
b. $\quad O_{h}$
c. $\quad O$
4.15 a. There are three possible orientations of the two blue faces.

If the blue faces are opposite each other, a $C_{3}$ axis connects the centers of the blue faces. This axis has 3 perpendicular $C_{2}$ axes, and contains three vertical mirror places $\left(\boldsymbol{D}_{3 \boldsymbol{d}}\right)$.

If the blue faces share one vertex of the octahedron, a $C_{2}$ axis includes this vertex, and this axis includes two vertical mirror planes $\left(\boldsymbol{C}_{\mathbf{2 v}}\right)$.

The third possibility is for the blue faces to share an edge of the octahedron. In this case, a $C_{2}$ axis bisects this shared edge, and includes two vertical mirror planes $\left(\boldsymbol{C}_{2 v}\right)$.
b. There are three unique orientations of the three blue faces.

If one blue face is arranged to form edges with each of the two remaining blue faces, the only symmetry operations are identity and a single mirror plane $\left(\boldsymbol{C}_{\boldsymbol{s}}\right)$.

If the three blue faces are arranged such that a single blue face shares an edge with one blue face, but only a vertex with the other blue face, the only symmetry operation is a mirror plane that passes through the center of the blue faces, and the point group is $\boldsymbol{C}_{\boldsymbol{s}}$.

If the three blue faces each share an edge with the same yellow face, a $C_{3}$ axis emerges from the center of this yellow face, and this axis includes three vertical mirror planes $\left(\boldsymbol{C}_{\mathbf{3 v}}\right)$.
c. If there are four different colors, and each pair of opposite faces has the identical color, the only symmetry operations are identity and inversion $\left(\boldsymbol{C}_{\boldsymbol{i}}\right)$.
4.16 Four point groups are represented by the symbols of the chemical elements. Most symbols have a single mirror in the plane of the symbol $\left(C_{s}\right)$, for example, Cs! Two symbols have $D_{2 h}$ symmetry (H, I), and two more (, S ) have $C_{2 h}$. Seven exhibit $C_{2 v}$ symmetry ( $B, C, K, V, Y, W, U$ ). In some cases, the choice of font may affect the point group. For example, the symbol for nitrogen may have $C_{2 h}$ in a sans serif font ( ) but otherwise $C_{s}(\mathrm{~N})$. The symbol of oxygen has $D_{\infty h}$ symmetry if shown as a circle but $D_{2 h}$ if oval.
4.17
a. on-deck circle $D_{\infty h}$
b. batter's box $\quad D_{2 h}$
c. cap $C_{s}$
e. home plate
$C_{2 v} \square$
f. baseball
$D_{2 d}($ see Figure 4.1)
d. bat
$C_{\infty \nu}$
4.18
a. $\quad D_{2 h}$
b. $\quad C_{2 v}$
c. $\quad C_{2 v}$
d. $\quad D_{4 h}$
e. $\quad C_{5 h}$
f. $\quad C_{2 v}$
g. $\quad D_{2 h}$
h. $\quad D_{4 h}$
i. $\quad C_{2}$
4.19 $\mathrm{SNF}_{3}$



Symmetry Operations:

after $E$

after $C_{3}$

after $\sigma_{v}(x z)$

Matrix Representations (reducible):

$$
E:\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \quad C_{3}:\left[\begin{array}{ccc}
\cos \frac{2 \pi}{3} & -\sin \frac{2 \pi}{3} & 0 \\
\sin \frac{2 \pi}{3} & \cos \frac{2 \pi}{3} & 0 \\
0 & 0 & 1
\end{array}\right]=\left[\begin{array}{ccc}
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\
\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\
0 & 0 & 1
\end{array}\right] \quad \sigma_{v}(x z):\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{array}\right]
$$

Characters of Matrix Representations:

Block Diagonalized Matrices:
$\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & {[1]}\end{array}\right] \quad\left[\begin{array}{ccc}{\left[\begin{array}{cc}-\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \sqrt{3} & -\frac{1}{2}\end{array}\right]} & 0 \\ 0 \\ 0 & 0 & {[1]}\end{array}\right]$

Irreducible Representations:

| $E$ | $2 C_{3}$ | $3 \sigma_{v}$ | Coordinates Used |
| :---: | :---: | :---: | :--- |
| 2 | -1 | 0 | $(x, y)$ |
| 1 | 1 | 1 | $z$ |

Character Table:

| $C_{3 v}$ | $E$ | $2 C_{3}$ | $3 \sigma_{v}$ | Matching Functions |  |
| :--- | :---: | :---: | ---: | :--- | :--- |
| $A_{1}$ | 1 | 1 | 1 | $z$ | $x^{2}+y^{2}, z^{2}$ |
| $A_{2}$ | 1 | 1 | -1 | $R_{z}$ |  |
| $E$ | 2 | -1 | 0 | $(x, y),\left(R_{x}, R_{y}\right)$ | $\left(x^{2}-y^{2}, x y\right)(x z, y z)$ |

$4.20 \quad$ a. $\quad C_{2 h}$ molecules have $E, C_{2}, i$, and $\sigma_{h}$ operations.

b. \(\quad\left[$$
\begin{array}{rll}1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1\end{array}
$$\right]\left[$$
\begin{array}{rrr}-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1\end{array}
$$\right]\left[\begin{array}{rrr}-1 \& 0 \& 0 <br>
0 \& -1 \& 0 <br>

0 \& 0 \& -1\end{array}\right]\left[\right.\)| $\sigma_{h}:$ |  |
| ---: | :---: | ---: |
| $\left.\begin{array}{rrr}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1\end{array}\right]$ |  |

c. These matrices can be block diagonalized into three $1 \times 1$ matrices, with the representations shown in the table.
$\begin{array}{llllcl} & \chi(E) & \chi\left(C_{2}\right) & \chi(i) & \chi\left(\sigma_{h}\right) & \\ B_{u} & 1 & -1 & -1 & 1 & \text { from the } x \text { and } y \text { coefficients } \\ A_{u} & 1 & 1 & -1 & -1 & \text { from the } z \text { coefficients }\end{array}$
The total is $\Gamma=2 B_{u}+A_{u}$.
d. Multiplying $B_{u}$ and $A_{u}$ :
$1 \times 1+(-1) \times 1+(-1) \times(-1)+1 \times(-1)=0$, proving they are orthogonal.
$4.21 \quad$ a. $\quad D_{2 h}$ molecules have $E, C_{2}(z), C_{2}(y), C_{2}(x), i, \sigma(x y), \sigma(x z)$, and $\sigma(y z)$ operations.

b. $\quad E:\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$
$C_{2}(z):\left[\begin{array}{rrr}-1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1\end{array}\right]$
$C_{2}(y):\left[\begin{array}{rrr}-1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1\end{array}\right]$
$C_{2}(x):\left[\begin{array}{rrr}1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1\end{array}\right]$
$i:\left[\begin{array}{rrr}-1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1\end{array}\right]$
$\sigma(x y):\left[\begin{array}{rrr}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1\end{array}\right]$
$\sigma(x z):\left[\begin{array}{rrr}-1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1\end{array}\right] \quad \sigma(y z):\left[\begin{array}{rrr}-1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$
c.

|  | $E$ | $C_{2}(z)$ | $C_{2}(y)$ | $C_{2}(x)$ | $i$ | $\sigma(x y)$ | $\sigma(x z)$ | $\sigma(y z)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\chi$ | 3 | -1 | -1 | -1 | -3 | 1 | 1 | 1 |
| $\Gamma$ |  |  |  |  |  |  |  |  |

d.

| -1 | -1 | 1 | -1 | 1 | 1 | -1 | matching $B_{3 u}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| -1 | 1 | -1 | -1 | 1 | -1 | 1 | matching $B_{2 u}$ |
| 1 | -1 | -1 | -1 | -1 | 1 | 1 | matching $B_{1 u}$ |

e. $\quad \Gamma_{1} \times \Gamma_{2}=1 \times 1+(-1) \times(-1)+(-1) \times 1+1 \times(-1)+(-1) \times(-1)+1 \times 1$
$+1 \times(-1)+(-1) \times 1=0$
$\Gamma_{1} \times \Gamma_{3}=1 \times 1+(-1) \times 1+(-1) \times(-1)+1 \times(-1)+(-1) \times(-1)+1 \times(-1)$
$+1 \times 1+(-1) \times 1=0$
$\Gamma_{2} \times \Gamma_{3}=1 \times 1+(-1) \times 1+1 \times(-1)+(-1) \times(-1)+(-1) \times(-1)+1 \times(-1)$
$+(-1) \times 1+1 \times 1=0$
4.22 a. $h=8$ (the total number of symmetry operations)
b. $\quad A_{1} \times E=1 \times 2+2 \times 1 \times 0+1 \times(-2)+2 \times 1 \times 0+2 \times 1 \times 0=0$
$A_{2} \times E=1 \times 2+2 \times 1 \times 0+1 \times(-2)+2 \times(-1) \times 0+2 \times(-1) \times 0=0$
$B_{1} \times E=1 \times 2+2 \times(-1) \times 0+1 \times(-2)+2 \times 1 \times 0+2 \times(-1) \times 0=0$
$B_{2} \times E=1 \times 2+2 \times(-1) \times 0+1 \times(-2)+2 \times(-1) \times 0+2 \times 1 \times 0=0$
c. $\quad E: 4+2 \times 0+4+2 \times 0+2 \times 0=8$
$A_{1}: 1+2 \times 1+1+2 \times 1+2 \times 1=8$
$A_{2}: 1+2 \times 1+1+2 \times 1+2 \times 1=8$
$B_{1}: 1+2 \times 1+1+2 \times 1+2 \times 1=8$
$B_{2}: 1+2 \times 1+1+2 \times 1+2 \times 1=8$
d. $\quad \Gamma_{1}=2 A_{1}+B_{1}+B_{2}+E$ :

$$
\begin{aligned}
& A_{1}: 1 / 8[1 \times 6+2 \times 1 \times 0+1 \times 2+2 \times 1 \times 2+2 \times 1 \times 2]=2 \\
& A_{2}: 1 / 8[1 \times 6+2 \times 1 \times 0+1 \times 2+2 \times(-1) \times 2+2 \times(-1) \times 2]=0 \\
& B_{1}: 1 / 8[1 \times 6+2 \times(-1) \times 0+1 \times 2+2 \times 1 \times 2+2 \times(-1) \times 2]=1 \\
& B_{2}: 1 / 8[1 \times 6+2 \times(-1) \times 0+1 \times 2+2 \times(-1) \times 2+2 \times 1 \times 2]=1 \\
& E: 1 / 8[2 \times 6+2 \times 0 \times 0+(-2) \times 2+2 \times 0 \times 2+2 \times 0 \times 2]=1 \\
& \Gamma_{2}=3 A_{1}+2 A_{2}+B_{1}: \\
& A_{1}: 1 / 8[1 \times 6+2 \times 1 \times 4+1 \times 6+2 \times 1 \times 2+2 \times 1 \times 0]=3 \\
& A_{2}: 1 / 8[1 \times 6+2 \times 1 \times 4+1 \times 6+2 \times(-1) \times 2+2 \times(-1) \times 0]=2 \\
& B_{1}: 1 / 8[1 \times 6+2 \times(-1) \times 4+1 \times 6+2 \times 1 \times 2+2 \times(-1) \times 0]=1 \\
& B_{2}: 1 / 8[1 \times 6+2 \times(-1) \times 4+1 \times 6+2 \times(-1) \times 2+2 \times 1 \times 0]=0 \\
& E: 1 / 8[2 \times 6+2 \times 0 \times 4+(-2) \times 6+2 \times 0 \times 2+2 \times 0 \times 0]=0
\end{aligned}
$$

$4.23 \quad C_{3 v}$
$\Gamma_{1}=3 A_{1}+A_{2}+E:$

$$
\begin{aligned}
& A_{1}: 1 / 6[1 \times 6+2 \times 1 \times 3+3 \times 1 \times 2]=3 \\
& A_{2}: 1 / 6[1 \times 6+2 \times 1 \times 3+3 \times(-1) \times 2]=1 \\
& E: 1 / 6[2 \times 6+2 \times(-1) \times 3+3 \times 0 \times 2]=1 \\
& \Gamma_{2}=A_{2}+E:
\end{aligned}
$$

$$
A_{1}: 1 / 6[1 \times 5+2 \times 1 \times(-1)+3 \times 1 \times(-1)]=0
$$

$$
A_{2}: 1 / 6[1 \times 5+2 \times 1 \times(-1)+3 \times(-1) \times(-1)]=1
$$

$$
E: 1 / 6[2 \times 5+2 \times(-1) \times(-1)+3 \times 0 \times(-1)]=2
$$

$$
O_{h}
$$

$$
\Gamma_{3}=A_{1 g}+E_{g}+T_{1 u}:
$$

$$
\begin{aligned}
& A_{1 g}: 1 / 48[6+0+0+12+6+0+0+0+12+12]=1 \\
& A_{2 g}: 1 / 48[6+0+0-12+6+0+0+0+12-12]=0 \\
& E_{g}: 1 / 48[12+0+0+0+12+0+0+0+24+0]=1 \\
& T_{1 g}: 1 / 48[18+0+0+12-6+0+0+0-12-12]=0 \\
& T_{2 g}: 1 / 48[18+0+0-12-6+0+0+0-12+12]=0 \\
& A_{1 u}: 1 / 48[6+0+0+12+6+0+0+0-12-12]=0 \\
& A_{2 u}: 1 / 48[6+0+0-12+6+0+0+0-12+12]=0 \\
& E_{u}: 1 / 48[12+0+0+0+12+0+0+0-24+0]=0 \\
& T_{1 u}: 1 / 48[18+0+0+12-6+0+0+0+12+12]=1 \\
& T_{2 u}: 1 / 48[18+0+0-12-6+0+0+0+12-12]=0
\end{aligned}
$$

4.24 The $d_{x y}$ characters match the characters of the $B_{2 g}$ representation:

4.25 Chiral: 4.5: $\mathrm{O}_{2} \mathrm{~F}_{2},\left[\mathrm{Cr}\left(\mathrm{C}_{2} \mathrm{O}_{4}\right)_{3}\right]^{3-}$ 4.6: none 4.7: screw, Wilkins Micawber 4.8: recycle symbol 4.9: set of three wind turbine blades, Flying Mercury sculpture, coiled spring
4.26
a. Point group: $C_{4 v}$

| $C_{4 v}$ | $E$ | $2 C_{4}$ | $C_{2}$ | $2 \sigma_{v}$ | $2 \sigma_{d}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 18 | 2 | -2 | 4 | 2 |  |
| $A_{1}$ | 1 | 1 | 1 | 1 | 1 | $z$ |
| $A_{2}$ | 1 | 1 | 1 | -1 | -1 | $R_{z}$ |
| $B_{1}$ | 1 | -1 | 1 | 1 | -1 |  |
| $B_{2}$ | 1 | -1 | 1 | -1 | 1 |  |
| $E$ | 2 | 0 | -2 | 0 | 0 | $(x, y),\left(R_{x}, R_{y}\right)$ |

b. $\quad \Gamma=4 A_{1}+A_{2}+2 B_{1}+B_{2}+5 E$
c. $\quad$ Translation: $A_{1}+E \quad($ match $x, y$, and $z)$

Rotation: $A_{2}+E \quad\left(\right.$ match $R_{x}, R_{y}$, and $R_{z}$ )
Vibration: all that remain: $3 A_{1}+2 B_{1}+B_{2}+3 E$
d. The character for each symmetry operation for the $\mathrm{Xe}-\mathrm{O}$ stretch is +1 . This corresponds to the $A_{1}$ irreducible representation, which matches the function $z$ and is therefore IR-active.

4.27 For $\mathrm{SF}_{6}$, the axes of the sulfur should point at three of the fluorines. The fluorine axes can be chosen in any way, as long as one from each atom is directed toward the sulfur atom. There are seven atoms with three axes each, for a total of 21 .

| $O_{h}$ | $E$ | $8 C_{3}$ | $6 C_{2}$ | $6 C_{4}$ | $3 C_{2}$ | $i$ | $6 S_{4}$ | $8 S_{6}$ | $3 \sigma_{h}$ | $6 \sigma_{d}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 21 | 0 | -1 | 3 | -3 | -3 | -1 | 0 | 5 | 3 |  |  |
| $T_{1 u}$ | 3 | 0 | -1 | 1 | -1 | -3 | -1 | 0 | 1 | 1 | $(x, y, z)$ |  |
| $T_{1 g}$ | 3 | 0 | -1 | 1 | -1 | 3 | 1 | 0 | -1 | -1 | $\left(R_{x}, R_{y}, R_{z}\right)$ |  |
| $A_{1 g}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |  |
| $A_{2 u}$ | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 |  | $\left(2 z^{2}-x^{2}-y^{2}\right.$, |
| $E_{g}$ | 2 | -1 | 0 | 0 | 2 | 2 | 0 | -1 | 2 | 0 |  | $\left.x^{2}-y^{2}\right)$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| $T_{2 u}$ | 3 | 0 | 1 | -1 | -1 | -3 | 1 | 0 | 1 | -1 |  | $(x y, x z, y z)$ |
| $T_{2 g}$ | 3 | 0 | 1 | -1 | -1 | 3 | -1 | 0 | -1 | 1 |  |  |

Reduction of $\Gamma$ gives $\Gamma=3 T_{1 u}+T_{1 g}+A_{1 g}+E_{g}+T_{2 g}+T_{2 u} \cdot T_{1 u}$ accounts for translation and also infrared active vibrational modes. $T_{\text {lg }}$ is rotation. The remainder are infrared-inactive vibrations.
a. $\quad$ cis- $\mathrm{Fe}(\mathrm{CO})_{4} \mathrm{Cl}_{2}$ has $C_{2 v}$ symmetry.

The vectors for CO stretching have the representation $\Gamma$ :

| $C_{2 v}$ | $E$ | $C_{2}$ | $\sigma_{v}(x z)$ | $\sigma_{v}{ }^{\prime}(y z)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 4 | 0 | 2 | 2 |  |
| $A_{1}$ | 1 | 1 | 1 | 1 | $z$ |
| $A_{2}$ | 1 | 1 | -1 | -1 |  |
| $B_{1}$ | 1 | -1 | 1 | -1 | $x$ |
| $B_{2}$ | 1 | -1 | -1 | 1 | $y$ |

$$
\begin{aligned}
& n\left(A_{1}\right)=1 / 4[4 \times 1+0 \times 1+2 \times 1+2 \times 1]=2 \\
& n\left(A_{2}\right)=1 / 4[4 \times 1+0 \times 1+2 \times(-1)+2 \times(-1)]=0 \\
& n\left(B_{1}\right)=1 / 4[4 \times 1+0 \times(-1)+2 \times 1+2 \times(-1)]=1 \\
& n\left(B_{2}\right)=1 / 4[4 \times 1+0 \times(-1)+2 \times(-1)+2 \times 1]=1 \\
& \Gamma=2 A_{1}+B_{1}+B_{2}, \text { all four IR active. }
\end{aligned}
$$

b. trans- $\mathrm{Fe}(\mathrm{CO})_{4} \mathrm{Cl}_{2}$ has $D_{4 h}$ symmetry.


| $D_{4 h}$ | $E$ | $2 C_{4}$ | $C_{2}$ | $2 C_{2}{ }^{\prime}$ | $2 C_{2}{ }^{\prime \prime}$ | $i$ | $2 S_{4}$ | $\sigma_{h}$ | $2 \sigma_{v}$ | $2 \sigma_{d}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\Gamma$ | 4 | 0 | 0 | 2 | 0 | 0 | 0 | 4 | 2 | 0 |  |
| $A_{2 u}$ | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | $z$ |
| $E_{u}$ | 2 | 0 | -2 | 0 | 0 | -2 | 0 | 2 | 0 | 0 | $(x, y)$ |

Omitting the operations that have zeroes in $\Gamma$ :
$n\left(A_{2 u}\right)=1 / 16[4 \times 1+2 \times 2 \times(-1)+4 \times(-1)+2 \times 2 \times 1]=0$
$n\left(E_{u}\right)=1 / 16[4 \times 2+2 \times 2 \times 0+4 \times 2+2 \times 2 \times 0]=1$ (IR active)
Note: In checking for IR-active bands, it is only necessary to check the irreducible representations having the same symmetry as $x, y$, or $z$, or a combination of them.
c. $\quad \mathrm{Fe}(\mathrm{CO})_{5}$ has $D_{3 h}$ symmetry.

The vectors for $\mathrm{C}-\mathrm{O}$ stretching have the following representation $\Gamma$ :

| $D_{3 h}$ | $E$ | $2 C_{3}$ | $3 C_{2}$ | $\sigma_{h}$ | $2 S_{3}$ | $3 \sigma_{\nu}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 5 | 2 | 1 | 3 | 0 | 3 |  |
| $E^{\prime}$ | 2 | -1 | 0 | 2 | -1 | 0 | $(x, y)$ |
| $A_{2}{ }^{\prime \prime}$ | 1 | 1 | -1 | -1 | -1 | 1 | $z$ |

$$
n\left(E^{\prime}\right)=1 / 12[(5 \times 2)+(2 \times 2 \times-1)+(3 \times 2)]=1
$$

$$
n\left(A_{2}{ }^{\prime \prime}\right)=1 / 12[(5 \times 1)+(2 \times 2 \times 1)+(3 \times 1 \times-1)+(3 \times-1)+(3 \times 3 \times 1)]=1
$$

There are two bands, one matching $E^{\prime}$ and one matching $A_{2}{ }^{\prime \prime}$. These are the only irreducible representations that match the coordinates $x, y$, and $z$.
4.29 In 4.28a, the symmetries of the CO stretching vibrations of cis- $\mathrm{Fe}(\mathrm{CO})_{4} \mathrm{Cl}_{2}\left(C_{2 v}\right.$ symmetry) are determined as $2 A_{1}+B_{1}+B_{2}$. Each of these representations matches Raman-active functions: $A_{1}$ $\left(x^{2}, y^{2}, z^{2}\right) ; A_{2}(x y), B_{1}(x z)$; and $B_{2}(y z)$, so all are Raman-active.

In 4.28b, the symmetries of the CO stretching vibrations of trans $-\mathrm{Fe}(\mathrm{CO})_{4} \mathrm{Cl}_{2}\left(D_{4 h}\right.$ symmetry) are $A_{1 g}+B_{1 g}+E_{u}$. Only $A_{1 g}\left(x^{2}+y^{2}, z^{2}\right)$ and $B_{1 g}\left(x^{2}-y^{2}\right)$ match Raman active functions; this complex exhibits two Raman-active CO stretching vibrations.

In 4.28c, the symmetries of the CO stretching vibrations of $\mathrm{Fe}(\mathrm{CO})_{5}\left(D_{3 h}\right.$ symmetry) are $2 A_{1}{ }^{\prime}+$ $E^{\prime}+A_{2}{ }^{\prime \prime}$. Only $A_{1}{ }^{\prime}\left(x^{2}+y^{2}, z^{2}\right)$ and $E^{\prime}\left(x^{2}-y^{2}, x y\right)$ match Raman-active functions; this complex exhibits four Raman-active CO stretching vibrations.
4.30 a. The point group is $C_{2 h}$.
b. Using the $\mathrm{Si}-\mathrm{I}$ bond vectors as a basis generates the representation:

| $C_{2 h}$ | $E$ | $C_{2}$ | $i$ | $\sigma_{h}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 4 | 0 | 0 | 0 |  |  |
| $A_{g}$ | 1 | 1 | 1 | 1 |  | $x^{2}+y^{2}, z^{2}$ |
| $B_{g}$ | 1 | -1 | 1 | -1 |  | $x z, y z$ |
| $A_{u}$ | 1 | 1 | -1 | -1 | $z$ |  |
| $B_{u}$ | 1 | -1 | -1 | 1 | $x, y$ |  |
| $=A_{g}+B_{g}+A_{u}+B_{u}$ |  |  |  |  |  |  |

The $A_{u}$ and $B_{u}$ vibrations are infrared active.
c. The $A_{g}$ and $B_{g}$ vibrations are Raman active.

### 4.31 trans isomer $\left(D_{4 h}\right)$ :


cis isomer $\left(C_{2 v}\right)$ :


The simplest approach is to consider if the number of infrared-active I-O stretches is different for these structures. (Alternatively, one could also determine the number of IR-active I-F stretches, a slightly more complicated task.)
trans:

| $D_{4 h}$ | $E$ | $2 C_{4}$ | $C_{2}$ | $2 C_{2}{ }^{\prime}$ | $2 C_{2}{ }^{\prime \prime}$ | $i$ | $2 S_{4}$ | $\sigma_{h}$ | $2 \sigma_{v}$ | $2 \sigma_{d}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 2 |  |
| $A_{1 g}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |
| $A_{2 u}$ | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | $z$ |

There is only a single IR-active I-O stretch (the antisymmetric stretch), $A_{2 u}$. cis:

| $C_{2 v}$ | $E$ | $C_{2}$ | $\sigma(x z)$ | $\sigma(y z)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 2 | 0 | 2 | 0 |  |
| $A_{1}$ | 1 | 1 | 1 | 1 | $z$ |
| $B_{1}$ | 1 | -1 | 1 | -1 | $x$ |

There are two IR-active I-O stretches, the $A_{1}$ and $B_{1}$ (symmetric and antisymmetric). Infrared spectra should therefore be able to distinguish between these isomers. (Reversing the $x$ and $y$ axes would give $A_{1}+B_{2}$. Because $B_{2}$ matches $y$, it would also represent an IR-active vibration.)

Because these isomers would give different numbers of IR-active absorptions, infrared spectra should be able to distinguish between them. The reference provides detailed IR data.
4.32 a. One way to deduce the number of Raman-active vibrations of $\mathrm{AsP}_{3}$ is to first determine the symmetries of all the degrees of freedom. This complex exhibits $C_{3}$ symmetry, with the $C_{3}$ axis emerging from the As atom. The $\chi(E)$ is 12 ; the
 $x, y$, and $z$ axes of the four atoms do not shift when the identity operation is carried out. Only the As atom contributes to the character of the $C_{3}$ transformation matrix; the P atoms shift during rotation about the $C_{3}$ axis. The general transformation matrix for rotation about the $z$ axis (Section 4.3.3) affords 0 as $\chi\left(C_{3}\right)$ for $\theta=120^{\circ}$. The As atom and one P atom do not shift when a $\sigma_{v}$ reflection is carried out, and $\chi\left(\sigma_{v}\right)=2$ (see the $\sigma_{v(x z)}$ transformation matrix in Section 4.33 for the nitrogen atom of $\mathrm{NH}_{3}$ as a model of how the two unshifted atoms of $\mathrm{AsP}_{3}$ will contribute to the character of the $\sigma_{v}$ transformation matrix).

| $C_{3 v}$ | $E$ | $C_{3}$ | $3 \sigma_{v}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 12 | 0 | 2 |  |  |
| $A_{1}$ | 1 | 1 | 1 | $z$ | $x^{2}+y^{2}, z^{2}$ |
| $A_{2}$ | 1 | 1 | -1 | $R_{x}$ |  |
| $E$ | 2 | -1 | 0 | $(x, y),\left(R_{x}, R_{y}\right)$ | $\left(x^{2}-y^{2}, x y\right),(x z, y z)$ |

Reduction of the reducible representation affords $3 A_{1}+A_{2}+4 E$.
On the basis of the character table, the translational modes of $\mathrm{AsP}_{3}$ have the symmetries $A_{1}+E$, and the rotational modes have the symmetries $A_{2}+E$. The six vibrational modes of $\mathrm{AsP}_{3}$ subsequently have the symmetries $2 A_{1}+2 E$. These vibrational modes are Raman-active, and four absorptions are expected (and observed) since the sets of $E$ modes are degenerate. B. M. Cossairt, M.C. Diawara, C. C. Cummins, Science 2009, 323, 602 assigns the bands as: $313\left(a_{I}\right), 345(e), 428\left(a_{1}\right), 557(e) \mathrm{cm}^{-1}$.

Alternatively, the set of six bonds may be selected as the basis for a representation focused specifically on stretches of these bonds. This approach generates the following representation:

| $C_{3 \nu}$ | $E$ | $C_{3}$ | $3 \sigma_{\nu}$ |  |  |
| :---: | :---: | :---: | :---: | :--- | :--- |
| $\Gamma$ | 6 | 0 | 2 |  |  |

This representation reduces to $2 A_{1}+2 E$, the same result as obtained by first considering all degrees of freedom, then subtracting the translational and rotational modes.
b. $\quad \mathrm{As}_{2} \mathrm{P}_{2}$ exhibits $C_{2 v}$ symmetry, and (like $\mathrm{AsP}_{3}$ ) will have six vibrational modes ( $3 N-6$ ). Inspection of the $C_{2 v}$ character table indicates that all vibrational modes will be Raman active. Since each irreducible representation has a dimension of 1 , the number of Raman absorptions expected is 6 (that is, there will be no degenerate vibrational modes).

This prediction can be confirmed via deduction of the symmetries of the vibrational modes. As in part a, we will first determine the symmetries of all the degrees of freedom. The $\chi(E)$ of the transformation matrix is 12 . In $\mathrm{As}_{2} \mathrm{P}_{2}$, the $C_{2}$ axis does not pass through any atoms, and all four atoms shift upon rotation; $\chi\left(C_{2}\right)=0$. Two atoms do not shift upon reflection through each of the $\sigma_{v}$ planes. The contribution to the character of the transformation matrix for each of these unshifted atoms is 1 , and $\chi\left(\sigma_{v}(x z)\right)=\chi\left(\sigma_{v}{ }^{\prime}(y z)\right)=2$.

| $C_{2 v}$ | $E$ | $C_{2}$ | $\sigma_{v}(x z)$ | $\sigma_{v}{ }^{\prime}(y z)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 12 | 0 | 2 | 2 |  |  |
| $A_{1}$ | 1 | 1 | 1 | 1 | $z$ | $x^{2}, y^{2}, z^{2}$ |
| $A_{2}$ | 1 | 1 | -1 | -1 | $R_{z}$ | $x y$ |
| $B_{1}$ | 1 | -1 | 1 | -1 | $x, R_{y}$ | $x z$ |
| $B_{2}$ | 1 | -1 | -1 | 1 | $y, R_{x}$ | $y z$ |

Reduction of the reducible representation affords $4 A_{1}+2 A_{2}+3 B_{1}+3 B_{2}$.
On the basis of the above character table, the translational modes of $\mathrm{As}_{2} \mathrm{P}_{2}$ have the symmetries $A_{1}+B_{1}+B_{2}$, and the rotational modes have the symmetries $A_{2}+B_{1}+B_{2}$. The six anticipated Raman-active vibrational modes of $\mathrm{As}_{2} \mathrm{P}_{2}$ subsequently have the symmetries $3 A_{1}+A_{2}+B_{1}+B_{2}$.

As in part a, an alternative approach is to select the set of six bonds as the basis for a representation focused specifically on stretching vibrations. This approach generates the following representation:

| $C_{2 v}$ | $E$ | $C_{2}$ | $\sigma_{v}(x z)$ | $\sigma_{v}{ }^{\prime}(y z)$ |  |  |
| :---: | :--- | :--- | :---: | :---: | :--- | :--- |
| $\Gamma$ | 6 | 2 | 2 | 2 |  |  |

This representation reduces to $3 A_{1}+A_{2}+B_{1}+B_{2}$, the same result as obtained by first considering all degrees of freedom, then subtracting the translational and rotational modes.
c. The issue here is whether or not $\mathrm{P}_{4}\left(T_{d}\right)$ exhibits 4 Raman-active vibrations as does $\mathrm{AsP}_{3}$. We will first determine the symmetries of all the degrees of
 freedom. The $\chi(E)$ of the transformation matrix is 12 . Only one $\mathrm{P}_{4}$ atom is fixed upon rotation about each $C_{3}$ axis; $\chi\left(C_{3}\right)=0$. All four atoms are shifted upon application of the $C_{2}$ and $S_{4}$ axes; $\chi\left(C_{2}\right)=\chi\left(S_{4}\right)=0$. Two atoms do not shift upon reflection through each of the $\sigma_{d}$ planes. The contribution to the character of the transformation matrix for each of these unshifted atoms is 1 , and $\chi\left(\sigma_{d}\right)=2$.

| $T_{d}$ | $E$ | $8 C_{3}$ | $3 C_{2}$ | $6 S_{4}$ | $6 \sigma_{d}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 12 | 0 | 0 | 0 | 2 |  |  |
| $A_{1}$ | 1 | 1 | 1 | 1 | 1 |  | $x^{2}+y^{2}+z^{2}$ |
| $A_{2}$ | 1 | 1 | 1 | -1 | -1 |  |  |
| $E$ | 2 | -1 | 2 | 0 | 0 |  | $\left(2 z^{2}-x^{2}-y^{2}, x^{2}-y^{2}\right)$ |
| $T_{1}$ | 3 | 0 | -1 | 1 | -1 | $\left(R_{x}, R_{y}, R_{z}\right)$ | $(x y, x z, y z)$ |
| $T_{2}$ | 3 | 0 | -1 | -1 | 1 | $(x, y, z)$ |  |

Reduction of the reducible representation affords $A_{1}+E+T_{1}+2 T_{2}$. Since the symmetries of the translational modes and rotational modes are $T_{2}$ and $T_{1}$, respectively, the symmetries of the vibrational modes are $A_{1}+E+T_{2}$, all of these modes are Ramanactive so three Raman absorptions are expected for $\mathrm{P}_{4}$, and $\mathrm{P}_{4}$ could potentially be distinguished from $\mathrm{AsP}_{3}$ solely on the basis of the number of Raman absorptions.

The alternative approach, using the set of six $\mathrm{P}-\mathrm{P}$ bonds as the basis for a representation focused specifically on bond stretches, generates the following representation:

| $T_{d}$ | $E$ | $8 C_{3}$ | $3 C_{2}$ | $6 S_{4}$ | $6 \sigma_{d}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :--- | :--- |
| $\Gamma$ | 6 | 0 | 2 | 0 | 2 |  |  |

This representation reduces to $A_{1}+E+T_{2}$, the same result as obtained by first considering all degrees of freedom, then subtracting the translational and rotational modes.
4.33 The possible isomers are as follows, with the triphenylphosphine ligand in either the axial (A) or equatorial (B) sites.

(A) $\quad C_{3 v}$

(B) $\quad C_{2 v}$

Note that the triphenylphosphine ligand is approximated as a simple L ligand for the sake of the point group determination. Rotation about the $\mathrm{Fe}-\mathrm{P}$ bond in solution is expected to render the arrangement of the phenyl rings unimportant in approximating the symmetry of these isomers in solution. The impact of the phenyl rings would likely be manifest in the IR $v(\mathrm{CO})$ spectra of these isomers in the solid-state.

For $\mathbf{A}$, we consider each CO bond as a vector to deduce the expected number of carbonyl stretching modes. The irreducible representation is as follows:

| $C_{3 v}$ | $E$ | $C_{3}$ | $3 \sigma_{v}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 4 | 1 | 2 |  |  |
| $A_{1}$ | 1 | 1 | 1 | $z$ | $x^{2}+y^{2}, z^{2}$ |
| $A_{2}$ | 1 | 1 | -1 | $R_{x}$ |  |
| $E$ | 2 | -1 | 0 | $(x, y),\left(R_{x}, R_{y}\right)$ | $\left(x^{2}-y^{2}, x y\right),(x z, y z)$ |

Reduction of the reducible representation affords $2 A_{l}+E$. These stretching modes are IR-active and three $v(\mathrm{CO})$ absorptions are expected for $\mathbf{A}$.

For $\mathbf{B}$, a similar analysis affords the following irreducible representation:

| $C_{2 v}$ | $E$ | $C_{2}$ | $\sigma_{v}(x z)$ | $\sigma_{v}{ }^{\prime}(y z)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 4 | 0 | 2 | 2 |  |  |
| $A_{1}$ | 1 | 1 | 1 | 1 | $z$ | $x^{2}, y^{2}, z^{2}$ |
| $A_{2}$ | 1 | 1 | -1 | -1 | $R_{z}$ | $x y$ |
| $B_{1}$ | 1 | -1 | 1 | -1 | $x, R_{y}$ | $x z$ |
| $B_{2}$ | 1 | -1 | -1 | 1 | $y, R_{x}$ | $y z$ |

Reduction of the reducible representation affords $2 A_{1}+B_{1}+B_{2}$. These stretching modes are IRactive, and four $v(C O)$ absorptions are expected for $\mathbf{A}$.

The reported $v(\mathrm{CO})$ IR spectrum is consistent with formation of isomer $\mathbf{A}$, with the triphenylphosphine ligand in the axial site.
4.34 As in 4.33, we consider the triphenylphosphine ligand as a simple L group for point group determination. The point groups for isomers A, B, and C are as follows:

(A) $C_{2 v}$

(B) $D_{3 h}$

(C) $C_{s}$

For $\mathbf{A}$, the set of irreducible representations for the three CO stretching vibrational modes is $2 A_{1}$
$+B_{1}$. These modes are all IR-active in the $C_{2 v}$ character table, and three $v(\mathrm{CO})$ IR absorptions are expected for isomer $\mathbf{A}$.

For $\mathbf{B}$, the set of irreducible representations for the three CO stretching vibrational modes is $A_{1}{ }^{\prime}+$ $E^{\prime}$. Only the $E^{\prime}$ mode is IR-active in the $D_{3 h}$ point group, and one $v(\mathrm{CO}) \mathrm{IR}$ absorption is expected for isomer $\mathbf{B}$.

For $\mathbf{C}$, the set of irreducible representations for the three CO stretching vibrational modes is $2 A^{\prime}+A^{\prime \prime}$. These modes are all IR-active in the $C_{s}$ point group, and three $v(\mathrm{CO})$ IR absorptions are expected for isomer $\mathbf{C}$.

The single $v(\mathrm{CO})$ IR absorption reported for $\mathrm{Fe}(\mathrm{CO})_{3}\left(\mathrm{PPh}_{3}\right)_{2}$ supports the presence of the $D_{3 h}$ isomer $\mathbf{B}$.

The trans isomer B is reported in R. L. Keiter, E. A. Keiter, K. H. Hecker, C. A. Boecker, Organometallics, 1988, 7, 2466, and the authors observe splitting of the absorption associated with the $E^{\prime}$ mode in $\mathrm{CHCl}_{3}$. The forbidden $A_{1}{ }^{\prime}$ stretching mode was observed as a weak absorption.
4.35 The IR spectrum exhibits two $v(\mathrm{CO})$ absorptions. The two proposed metal carbonyl fragments will be considered independently for analysis.

$D_{4 h} \quad C_{4 v}$

The reducible representation for the four vectors associated with the CO bonds in the $C_{4 v}$ fragment is as follows:

| $C_{4 v}$ | $E$ | $2 C_{4}$ | $C_{2}$ | $2 \sigma_{v}$ | $2 \sigma_{d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 4 | 0 | 0 | 2 | 0 |

Reduction of this representation affords $A_{1}+B_{1}+E$. The $A_{1}$ and $E$ modes are IR-active, and a titanium complex with a square pyramidal titanium tetracarbonyl fragment is supported by the IR spectral data.

For the $D_{4 h}$ fragment, the reducible representation for the set of vectors associated with the CO bonds is:

| $D_{4 h}$ | $E$ | $2 C_{4}$ | $C_{2}$ | $2 C_{2}{ }^{\prime}$ | $2 C_{2}{ }^{\prime \prime}$ | $i$ | $2 S_{4}$ | $\sigma_{h}$ | $2 \sigma_{v}$ | $2 \sigma_{d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 4 | 0 | 0 | 2 | 0 | 0 | 0 | 4 | 2 | 0 |

Reduction of the reducible representation affords $A_{1 g}+B_{1 g}+E_{u}$. Only the $E$ modes are IR-active, and a complex with a square planar titanium tetracarbonyl fragment is expected to exhibit a single IR $v(\mathrm{CO})$ stretching absorption. This $D_{4 h}$ possibility can therefore be ruled out on the basis of the spectrum.
4.36 A reasonable product is the $C_{4 v}$ molecule $\mathrm{Mo}(\mathrm{CO})_{5}\left(\mathrm{P}(\mathrm{OPh})_{3}\right)$, with CO replacing a triphenylphosphite ligand.

The reducible representation for the five vectors associated with the CO bonds in this molecule is:


| $C_{4 v}$ | $E$ | $2 C_{4}$ | $C_{2}$ | $2 \sigma_{v}$ | $2 \sigma_{d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 5 | 1 | 1 | 3 | 1 |

Reduction of this representation affords $2 A_{1}+B_{1}+E$. The $A_{1}$ and $E$ modes are IR-active, and three IR $v(\mathrm{CO})$ stretching absorptions are expected. The reported IR spectrum features three strong $v(\mathrm{CO})$ absorptions, and one "very weak" absorption attributed to the forbidden $B_{1}$ mode in D. J. Darensbourg, T. L. Brown, Inorg. Chem., 1968, 7, 959.
4.37 I has $C_{2}$ symmetry, with a $C_{2}$ axis running right to left, perpendicular to the $\mathrm{Cl}, \mathrm{N}, \mathrm{Cl}, \mathrm{N}$ and $\mathrm{Cl}, \mathrm{P}, \mathrm{Cl}, \mathrm{P}$ faces.
II also has $C_{2}$ symmetry, with the same $C_{2}$ axis as I. (Lower left corner occupied by Cl , not C .)
III has only an inversion center and $C_{i}$ symmetry.
4.38 An example for each of the five possible point groups:

4.39 a. The $\mathrm{S}-\mathrm{C}-\mathrm{C}$ portion is linear, so the molecule has a $C_{3}$ axis along the line of these three atoms, three $\sigma_{v}$ planes through these atoms and an F atom on each end, but no other symmetry elements. $\boldsymbol{C}_{\mathbf{3}}$ v
b. The molecule has only an inversion center, so it is $\boldsymbol{C}_{\boldsymbol{i}}$. The inversion center is equivalent to an $S_{2}$ axis perpendicular to the average plane of the ring.
c. $\quad \mathrm{M}_{2} \mathrm{Cl}_{6} \mathrm{Br}_{4}$ is $\boldsymbol{C}$.
d. This complex has a $C_{3}$ axis, splitting the three N atoms and the three P atoms (almost as drawn), but no other symmetry elements. $\boldsymbol{C}_{3}$
e. The most likely isomer has the less electronegative Cl atoms in equatorial positions. Point group: $\boldsymbol{C}_{\mathbf{2 v}}$

4.40 The structures on the top row are $D_{2 d}$ (left) and $C_{s}$ (right). Those on the bottom row are $C_{2 h}$ (left) and $C_{4 v}$ (right).
4.41 a. $C_{3 v}$
b. $\quad D_{5 h}$
c. $\quad$ Square structure: $D_{2 d}$ (bottom ligand on lower left Re should be CO instead of L); Corner structure: $C_{s}$
d. $\quad D_{3 d}$
4.42 Web of Science and SciFinder Scholar should be helpful, but simply searching for these symmetries using a general Internet search should provide examples of these point groups. Some examples:
a. $\quad S_{6}: \quad \mathrm{Mo}_{2}\left(\mathrm{SC}_{6} \mathrm{H}_{2} \mathrm{Me}_{3}\right)_{6}$ (M. H. Chisholm, J. F. Corning, and J. C. Huffman, J. Am. Chem. Soc., 1983, 105, 5924)
$\mathrm{Mo}_{2}\left(\mathrm{NMe}_{2}\right)_{6}$ (M. H. Chisholm, R. A. Cotton, B. A. Grenz, W. W. Reichert, L. W. Shive, and B. R. Stults, J. Am. Chem. Soc., 1976, 98, 4469)
$\left[\mathrm{NaFe}_{6}(\mathrm{OMe})_{12}(\mathrm{dbm})_{6}\right]^{+}\left(\mathrm{dbm}=\right.$ dibenzoylmethane, $\left.\mathrm{C}_{6} \mathrm{H}_{5} \mathrm{COCCOC}_{6} \mathrm{H}_{5}\right)(\mathrm{F}$. L. Abbati, A. Cornia, A. C. Fabretti, A. Caneschi, and D. Garreschi, Inorg. Chem., 1998, 37, 1430)
b. $\quad T \quad \operatorname{Pt}\left(\mathrm{CF}_{3}\right)_{4}, \mathrm{C}_{44}$
c. $\quad I_{h} \quad \mathrm{C}_{20}, \mathrm{C}_{80}$
d. $\quad T_{h} \quad\left[\mathrm{Co}\left(\mathrm{NO}_{2}\right)_{6}\right]^{3-}, \mathrm{Mo}\left(\mathrm{NMe}_{2}\right)_{6}$

In addition to examples that can be found using Web of Science, SciFinder, and other Internet search tools, numerous examples of these and other point groups can be found in I. Hargittai and M. Hargittai, Symmetry Through the Eyes of a Chemist, as listed in the General References section.


[^0]:    * Incorrectly cited as problem 4.30 in first printing of text.

[^1]:    * Incorrectly cited as problem 3.41 in first printing of text.

