$$\Delta X(t) = \Delta X_0 cos(\omega t)$$

$$\Delta Y(t) = \Delta Y_0 cos(\omega t + \phi)$$

ΔX is a small oscillating perturbation that induces a response ΔY(t)

$$\Delta x(s) = \Delta X_0 \frac{s}{(s^2 + \omega^2)}$$

Take the Laplace transform ΔX(t)

$$\Delta y(s) = H(s)\Delta x(s)$$

y(s) is the Laplace transform of ΔY(t)
H(s) is called the "transfer function"

$$\Delta y(s) = \Delta X_0 H(s) \frac{s}{(s^2 + \omega^2)}$$

We don't know H(s) yet, but we know x(s)

Bode and Nyquist Plots

$$\Delta y(s) = \Delta X_0 H(s) \frac{s}{(s^2 + \omega^2)}$$

$$s^2 + \omega^2 = (s + i\omega)(s - i\omega)$$

Partial Fraction Expansion

$$\Delta y(s) = \frac{A}{(s - i\omega)} + \frac{A*}{(s + i\omega)} + \ldots$$

$$A = (s - i\omega)\frac{\Delta X_0 H(s) s}{(s + i\omega)(s - i\omega)} = \frac{\Delta X_0}{2} H(i\omega)$$

$$s \to i\omega$$

$$\Delta y(s) = \frac{\Delta X_0}{2}\left(\frac{H(i\omega)}{(s - i\omega)} + \frac{H*(i\omega)}{(s + i\omega)}\right)$$

$$H(i\omega) = |H(i\omega)| e^{i\phi}$$

$$H*(i\omega) = |H(i\omega)| e^{-i\phi}$$

$$\Delta Y(t) = \Delta X_0 |H(i\omega)| cos(\omega t + \phi)$$

$$H(i\omega) = |H(i\omega)| e^{i\phi}$$

$$\phi = tan^{-1}\left(\frac{ImH}{ReH}\right)$$

Bode Plots

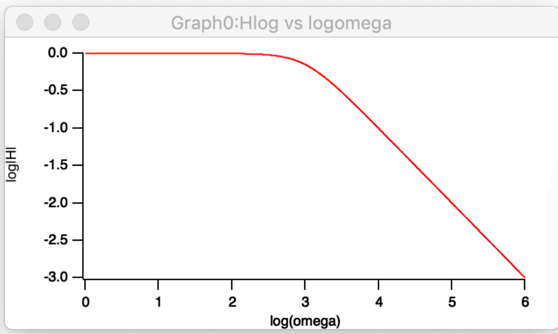
$$\Delta X(t) = \Delta X_0 cos(\omega t)$$

$$\Delta Y(t) = \Delta X_0 |H(i\omega)| cos(\omega t + \phi)$$

$$\phi = tan^{-1}\left(\frac{ImH}{ReH}\right)$$

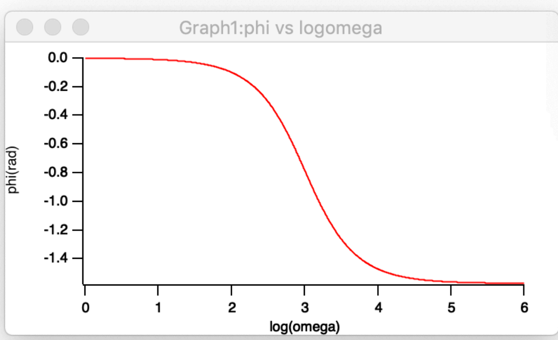
Frequency Plot:

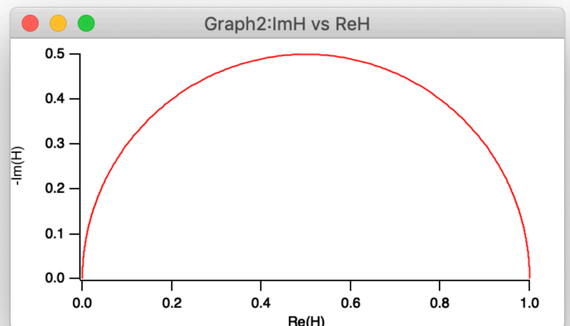
log(H) vs. log(ω)



Phase Shift Plot:

ϕ vs. log(ω)





Nyquist Plot:

-Im(H) vs. Re(H)

Bode Plots: First Example

$$\Delta X(t) = \Delta X_0 cos(\omega t)$$

$$\Delta Y(t) = \Delta X_0 |H(i\omega)| cos(\omega t + \phi)$$

$$\phi = tan^{-1}\left(\frac{ImH}{ReH}\right)$$

$$H(s) = \frac{K}{1 + s\tau}$$

$$H(i\omega) = \frac{K}{1 + i\omega\tau}$$

Bode Plots: First Example

$$\Delta X(t) = \Delta X_0 cos(\omega t)$$

$$\Delta Y(t) = \Delta X_0 |H(i\omega)| cos(\omega t + \phi)$$

$$\phi = tan^{-1}\left(\frac{ImH}{ReH}\right)$$

$$H(i\omega) = \frac{K}{1 + i\omega\tau} = \frac{K}{1 + \omega^2\tau^2}[1 - i\omega\tau]$$

$$ReH = \frac{K}{1 + \omega^2\tau^2}$$

$$|H(i\omega)| = \left[(ReH)^2 + (ImH)^2\right]^{\frac{1}{2}}$$

$$ImH = \frac{-K\omega\tau}{1 + \omega^2\tau^2}$$

$$\phi = tan^{-1}\left(\frac{ImH}{ReH}\right) = tan^{-1}(-\omega\tau)$$

Bode Plots: First Example

$$ReH = \frac{K}{1+\omega^2\tau^2} \qquad ImH = \frac{-K\omega\tau}{1+\omega^2\tau^2}$$

```
Make/N=1000/D logomega, omega, ReH, ImH, Hmag, Hlog, phi
Make/N=1000/D/C H
Variable/G Smax := 1
Variable/G tau := 1/1000
logomega := 6*x/1000
omega := 10^(logomega)
H := Smax/(1 + cmplx(0,1)*omega*tau)
Hmag := sqrt(magsqr(H))
Hlog := log(Hmag)
phi := imag(r2polar(H))
display Hlog vs logomega
Label left "log|H|"
Label bottom "log(omega)"
display phi vs logomega
Label left "phi(rad)"
Label bottom "log(omega)"
ImH := -imag(H)
ReH := real(H)
display ImH vs ReH
Label left "-Im(H)"
Label bottom "Re(H)"
```