Diffusion Equations in One Dimension R. Corn 11/18/03

A. Fick's Laws of Diffusion

For a species A in solution, its concentration becomes a function of both space and time:

$$C_A(x,t)$$

$$J_A = -D_A \frac{\partial C_A}{\partial x}$$ Fick's First Law of Diffusion

$$\frac{\partial C_A}{\partial t} = -\frac{\partial J_A}{\partial x} = D_A \frac{\partial^2 C_A}{\partial x^2}$$ Fick's Second Law of Diffusion

D_A is the diffusion constant for species A.

B. Instantaneous Plane Source Solution

Boundary Conditions:

$$C_A(x = 0, t = 0) =$$
$$C_A(x \neq 0, t = 0) = 0$$

Solution:

$$C_A = \frac{N_A}{2(\pi D_A t)^{1/2}} \exp\left[\frac{-x^2}{4 D_A t}\right]$$

where N_A is the total number of molecules:

$$N_A = \int_{-\infty}^{+\infty} C_A \, dx$$

C. Spreading Probability

What is the probability that a molecule has diffused a distance x in time t?

We define *p(x)dx* is the probability that a molecule has diffused to a region between *x* and *x* + *dx*:

$$p(x)dx = \frac{C_A}{N_A}dx = \frac{1}{2(\pi D_A t)^{1/2}} \exp\left[\frac{-x^2}{4D_A t}\right]dx$$

The mean square distance diffused by a molecule in time t is:

$$\langle x^2 \rangle = \int_{-\infty}^{+\infty} x^2 p(x)dx = 2D_A t$$

$$\sqrt{\langle x^2 \rangle} = \sqrt{2D_A t}$$