Angular Momentum Coupling

Orbital Angular Momentum, L:

$$L^2|l,m_l> = \hbar^2 l(l+1)|l,m_l>$$

$$L_z|l,m_l> = \hbar m_l|l,m_l>$$

Spin Angular Momentum, S:

$$S^2|s,m_s> = \hbar^2 s(s+1)|s,m_s>$$

$$S_z|s,m_s> = \hbar m_s|s,m_s>$$

Total Angular Momentum, J:

$$J^2|j,m_j> = \hbar^2 j(j+1)|j,m_j>$$

$$J_z|j,m_j> = \hbar m_j|j,m_j>$$

$$j = (l+s, l+s-1 \ldots |l-s|)$$

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

$$J^2 = L^2 + S^2 + 2\mathbf{L}\cdot\mathbf{S}$$

$$\mathbf{L}\cdot\mathbf{S} = L_x S_x + L_y S_y + L_z S_z$$

$$\mathbf{L}\cdot\mathbf{S} = L_z S_z + \frac{1}{2}(L_+ S_- + L_- S_+)$$

$$J^2 = L^2 + S^2 + 2L_z S_z + L_+ S_- + L_- S_+$$

Term Symbol: $^{2S+1}L_J$

Raising and Lowering Operators

$$L_\pm = L_x \pm iL_y$$

$$S_\pm = S_x \pm iS_y$$

Dot Product Coupling

$$H_{int} = a\mathbf{J}\cdot\mathbf{S}$$

$$\mathbf{J}\cdot\mathbf{S} = J_xS_x + J_yS_y + J_zS_z$$

Raising and Lowering Operators:

$$J_+ = J_x + iJ_y \qquad S_+ = S_x + iS_y$$

$$J_- = J_x - iJ_y \qquad S_- = S_x - iS_y$$

$$J_+S_- + J_-S_+ = (J_x + iJ_y)(S_x - iS_y) + (J_x - iJ_y)(S_x + iS_y)$$

$$J_+S_- + J_-S_+ = 2(J_xS_x + J_yS_y)$$

$$\mathbf{J}\cdot\mathbf{S} = J_zS_z + \frac{1}{2}(J_+S_- + J_-S_+)$$

Two Identical Spins - j Coupling

$$H = H_0 + W$$

$$H_0 = -\,\omega_0 S_z^A - \omega_0 S_z^B$$

$$W = \frac{j}{\hbar}\mathbf{S}^A \cdot \mathbf{S}^B$$

$$W = \frac{j}{\hbar}\left(S_z^A S_z^B + \frac{1}{2}(S_+^A S_-^B + S_-^A S_+^B)\right)$$

$$S_+ = S_x + iS_y$$
$$S_- = S_x - iS_y$$

$$S_x^A S_x^B + S_y^A S_y^B = \frac{1}{2}\left(S_+^A S_-^B + S_-^A S_+^B\right)$$

$$S_-^A S_+^B \,|+-> = \hbar^2 \,|-+>$$

$$S_+^A S_-^B \,|-+> = \hbar^2 \,|+->$$

$$H|++> = \left(-\hbar\omega_0 + \frac{\hbar j}{4}\right)|++>$$

$$H|+-> = -\frac{\hbar j}{4}|+-> + \frac{\hbar j}{2}|-+>$$

$$H|-+> = -\frac{\hbar j}{4}|-+> + \frac{\hbar j}{2}|+->$$

$$H|--> = \left(\hbar\omega_0 + \frac{\hbar j}{4}\right)|-->$$

$$H = H_0 + W$$

$$\begin{bmatrix} -\hbar\omega_0 + \frac{\hbar j}{4} & 0 & 0 & 0 \\ 0 & -\frac{\hbar j}{4} & \frac{\hbar j}{2} & 0 \\ 0 & \frac{\hbar j}{2} & -\frac{\hbar j}{4} & 0 \\ 0 & 0 & 0 & \hbar\omega_0 + \frac{\hbar j}{4} \end{bmatrix}$$