The Electric Susceptibility, Dielectric Constant, and Complex Index of Refraction

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Electric Polarization: $\mathbf{P}(\omega) = \varepsilon_0 \chi(\omega) \mathbf{E}(\omega)$

Electric Displacement: $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$

$$\mathbf{D} = \varepsilon_0(1 + \chi) \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon_r(\omega)\mathbf{E} = \varepsilon\mathbf{E}$$

$\chi(\omega)$ = complex frequency dependent electric susceptibility

ε_0 = permittivity of free space
ε = permittivity
$\varepsilon_r(\omega)$ = relative permittivity or complex frequency dependent dielectric constant

$\chi = \chi' + i\chi''$

$\varepsilon_r(\omega) = 1 + \chi = (1 + \chi) + i\chi''$

EM Plane Wave: $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$

In free space: $k = \omega(\varepsilon_0\mu_0)^{1/2} = \omega /c$

c = speed of light
μ_0 = permeability of free space

In a dielectric: $k = \omega(\varepsilon\mu_0)^{1/2}\ k = \omega(\varepsilon_r \varepsilon_0\mu_0)^{1/2} = n\omega /c$

$$n(\omega) = (\varepsilon_r)^{1/2}$$

$n(\omega) = \eta + i\kappa$

$n(\omega)$ = complex index of refraction
η = (real) refractive index
κ = extinction coefficient

EM wave in the z direction:

$$E(z,t) = E_0 \exp\left(i\omega\left[\frac{nz}{c} - t\right] - \frac{\omega\kappa z}{c}\right)$$

Beer's Law: $I(z) = I_0 \exp(-Kz)$ $K = 2\omega\kappa/c$

K = Beer's Law absorption coefficient