

SHO: Dimensionless x and p Operators

$$\hat{x} = \sqrt{\frac{m\omega}{\hbar}} x$$

$$H = \hbar\omega \hat{H}$$

$$\hat{p} = \frac{p}{\sqrt{m\hbar\omega}}$$

$$\hat{H} = \frac{1}{2} (\hat{x}^2 + \hat{p}^2)$$

$$[\hat{x}, \hat{p}] = i$$

Hamiltonian

Commutator

Lowering Operator: $a = \frac{1}{\sqrt{2}} (\hat{x} + i\hat{p})$

$$a |n\rangle = \sqrt{n} |n-1\rangle$$

Raising Operator: $a^\dagger = \frac{1}{\sqrt{2}} (\hat{x} - i\hat{p})$

$$a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$$

Number Operator: $\hat{N} = a^\dagger a$

$$\hat{N} |n\rangle = a^\dagger a |n\rangle = n |n\rangle$$

Commutator: $[a, a^\dagger] = 1$