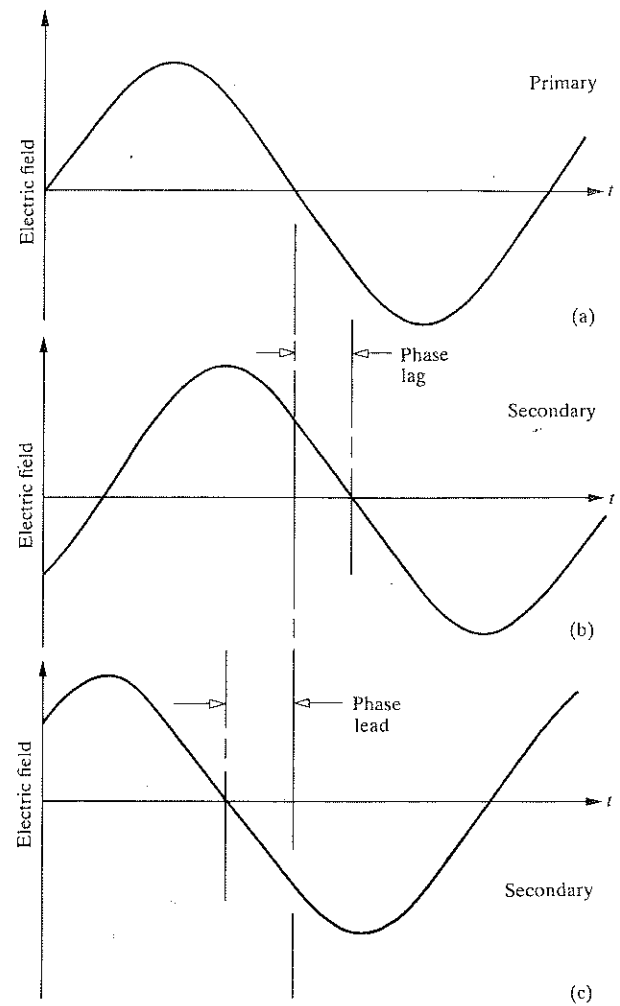


**Figure 4.9** A schematic representation of (a) amplitude and (b) phase lag versus driving frequency for a damped oscillator. The dashed curves correspond to decreased damping. The corresponding index of refraction is shown in (c).

mary (Fig. 4.10) by some amount between approximately  $90^\circ$  and  $180^\circ$ , and at frequencies above resonance, the lag ranges from about  $180^\circ$  to  $270^\circ$ . But a phase lag of  $\delta \geq 180^\circ$  is equivalent to a phase lead of  $360^\circ - \delta$ , [e.g.,  $\cos(\theta - 270^\circ) = \cos(\theta + 90^\circ)$ ]. This much can be seen on the right side of Fig. 4.9b.

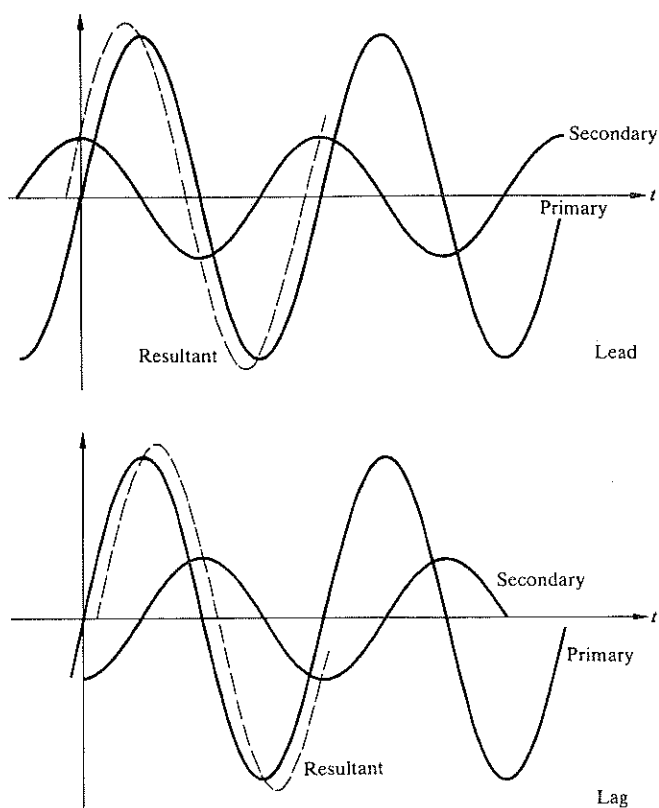
Within the transparent medium, the primary and secondary waves overlap and, depending on their amplitudes and relative phase, generate the net transmitted disturbance. Except for the fact that it is weakened by scattering, the primary wave travels into the material just as if it were traversing free space. By



**Figure 4.10** A primary wave (a) and two possible secondary waves. In (b) the secondary lags the primary—it takes longer to reach any given value. In (c) the secondary wave reaches any given value before (at an earlier time than) the primary; that is, it leads.

comparison to this free-space wave, which initiated the process, the resultant transmitted wave is phase shifted, and this phase difference is crucial.

When the secondary wave lags (or leads) the primary, the resultant transmitted wave must also lag (or lead) it by some amount (Fig. 4.11). This qualitative relationship will serve our purposes for the moment, although it should be noted that the phase of the resultant also depends on the amplitudes of the



**Figure 4.11** If the secondary leads the primary, the resultant will also lead it.

interacting waves [see Eq. (7.10)]. At frequencies below  $\omega_0$  the transmitted wave lags the free-space wave, whereas at frequencies above  $\omega_0$  it leads the free-space wave. For the special case in which  $\omega = \omega_0$  the secondary and primary waves are out-of-phase by  $180^\circ$ . The former works against the latter, so that the refracted wave is appreciably reduced in amplitude although unaffected in phase.

As the transmitted wave advances through the medium, scattering occurs over and over again. Light traversing the substance is progressively retarded (or advanced) in phase. Evidently, since the speed of the wave is the rate of advance of the condition of constant phase, a change in the phase should correspond to a change in the speed.

We now wish to show that a phase shift is indeed tantamount to a difference in phase velocity. In free space, the dis-

turbance at some point  $P$  may be written as

$$E_P(t) = E_0 \cos \omega t \quad (4.1)$$

If  $P$  is surrounded by a dielectric, there will be a cumulative phase shift  $\epsilon_P$ , which was built up as the wave moved through the medium to  $P$ . At ordinary levels of irradiance the medium will behave linearly, and the frequency in the dielectric will be the same as that in vacuum, even though the wavelength and speed may differ. Once again, but this time in the medium, the disturbance at  $P$  is

$$E_P(t) = E_0 \cos (\omega t - \epsilon_P) \quad (4.2)$$

where the subtraction of  $\epsilon_P$  corresponds to a phase lag. An observer at  $P$  will have to wait a longer time for a given crest to arrive when she is in the medium than she would have had to wait in vacuum. That is, if you imagine two parallel waves of the same frequency, one in vacuum and one in the material, the vacuum wave will pass  $P$  a time  $\epsilon_P/\omega$  before the other wave. Clearly then, a phase lag of  $\epsilon_P$  corresponds to a reduction in speed,  $v < c$  and  $n > 1$ . Similarly, a phase lead yields an increase in speed,  $v > c$  and  $n < 1$ . Again, the scattering process is a continuous one, and the cumulative phase shift builds as the light penetrates the medium. That is to say,  $\epsilon$  is a function of the length of dielectric traversed, as it must be if  $v$  is to be constant (see Problem 4.5). In the vast majority of situations encountered in Optics  $v < c$  and  $n > 1$ ; see Table 4.1. The important exception is the case of X-ray propagation where  $\omega > \omega_0$ ,  $v > c$ , and  $n < 1$ .

The overall form of  $n(\omega)$ , as depicted in Fig. 4.9c, can now be understood as well. At frequencies far below  $\omega_0$  the amplitudes of the oscillators and therefore of the secondary waves are very small, and the phase angles are approximately  $90^\circ$ . Consequently, the refracted wave lags only slightly, and  $n$  is only slightly greater than 1. As  $\omega$  increases, the secondary waves have greater amplitudes and lag by greater amounts. The result is a gradually decreasing wave speed and an increasing value of  $n > 1$ . Although the amplitudes of the secondary waves continue to increase, their relative phases approach  $180^\circ$  as  $\omega$  approaches  $\omega_0$ . Consequently, their ability to cause a further increase in the resultant phase lag diminishes. A turning point ( $\omega = \omega'$ ) is reached where the refracted wave begins to experience a decreasing phase lag and an increasing speed ( $dn/d\omega < 0$ ). That continues until  $\omega = \omega_0$ , whereupon the transmitted wave is appreciably reduced in amplitude but unaltered in phase and speed. At that point,  $n = 1$ ,  $v = c$ , and we are more or less at the center of the absorption band.

**TABLE 4.1** Approximate Indices of Refraction of Various Substances\*

Air	1.000 29
Ice	1.31
Water	1.333
Ethyl alcohol (C <sub>2</sub> H <sub>5</sub> OH)	1.36
Fused quartz (SiO <sub>2</sub> )	1.458 4
Carbon tetrachloride (CCl <sub>4</sub> )	1.46
Turpentine	1.472
Benzene (C <sub>6</sub> H <sub>6</sub> )	1.501
Plexiglass	1.51
Crown glass	1.52
Sodium chloride (NaCl)	1.544
Light flint glass	1.58
Polystyrene	1.59
Carbon disulfide (CS <sub>2</sub> )	1.628
Dense flint glass	1.66
Lanthanum flint glass	1.80
Zircon (ZrO <sub>2</sub> ·SiO <sub>2</sub> )	1.923
Fabulite (SrTiO <sub>3</sub> )	2.409
Diamond (C)	2.417
Rutile (TiO <sub>2</sub> )	2.907
Gallium phosphide	3.50

\*Values vary with physical conditions—purity, pressure, etc. These correspond to a wavelength of 589 nm

At frequencies just beyond  $\omega_0$  the relatively large-amplitude secondary waves lead; the transmitted wave is advanced in phase, and its speed exceeds  $c$  ( $n < 1$ ). As  $\omega$  increases, the whole scenario is played out again in reverse (with some asymmetry due to frequency-dependent asymmetry in oscillator amplitudes and scattering). At even higher frequencies the secondary waves, which now have very small amplitudes, lead by nearly  $90^\circ$ . The resulting transmitted wave is advanced very slightly in phase, and  $n$  gradually approaches 1.

The precise shape of a particular  $n(\omega)$  curve depends on the specific oscillator damping, as well as on the amount of absorption, which in turn depends on the number of oscillators participating.

A rigorous solution to the propagation problem is known as the *Ewald-Oseen Extinction Theorem*. Although the mathematical formalism, involving integro-differential equations, is far too complicated to treat here, the results are certainly of interest. It is found that the electron-oscillators generate an

electromagnetic wave having essentially two terms. One of these precisely cancels the primary wave within the medium. The other, and only remaining disturbance, moves through the dielectric at a speed  $v = c/n$  as the transmitted wave.\* *Henceforth we shall simply assume that a lightwave propagating through any substantive medium travels at a speed  $v \neq c$ .*

Apparently, any quantum-mechanical model we construct will somehow have to associate a wavelength with the photon. That's easily done mathematically via the expression  $p = h/\lambda$ , even if it's not clear at this point what is doing the waving. Still the wave nature of light seems inescapable; it will have to be infused into the theory one way or another. And once we have the idea of a photon-wavelength, it's natural to bring in the concept of relative phase. Thus *the index of refraction arises when the absorption and emission process advances or retards the phases of the scattered photons, even as they travel at speed  $c$ .*

## 4.3 Reflection

When a beam of light impinges on the surface of a transparent material, such as a sheet of glass, the wave "sees" a vast array of closely spaced atoms that will somehow scatter it. Remember that the wave may be  $\approx 500$  nm long, whereas the atoms and their separations ( $\approx 0.2$  nm) are thousands of times smaller. In the case of transmission through a dense medium, the scattered wavelets cancel each other in all but the forward direction, and just the ongoing beam is sustained. But that can only happen if there are no discontinuities. This is not the case at an interface between two different transparent media (such as air and glass), which is a jolting discontinuity. When a beam of light strikes such an interface, some light is always scattered backward, and we call this phenomenon **reflection**.

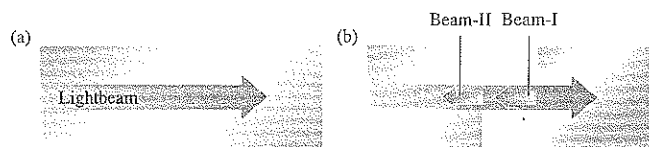
If the transition between two media is gradual—that is, if the dielectric constant (or the index of refraction) changes from that of one medium to that of the other over a distance of a wavelength or more—there will be very little reflection; the interface effectively vanishes. On the other hand, a transition from one medium to the other over a distance of  $1/4$  wavelength or less behaves very much like a totally discontinuous change.

\*For a discussion of the Ewald-Oseen theorem, see *Principles of Optics* by Born and Wolf, Section 2.4.2; this is heavy reading. Also look at Reali, "Reflection from dielectric materials," *Am. J. Phys.* **50**, 1133 (1982).

## Internal and External Reflection

Imagine that light is traveling across a large homogeneous block of glass (Fig. 4.12). Now, suppose that the block is sheared in half perpendicular to the beam. The two segments are then separated, exposing the smooth flat surfaces depicted in Fig. 4.12b. Just before the cut was made, there was no light-wave traveling to the left inside the glass—we know the beam only advances. Now there must be a wave (beam-I) moving to the left, reflected from the surface of the right-hand block. The implication is that a region of scatterers on and beneath the exposed surface of the right-hand block is now “unpaired,” and the backward radiation they emit can no longer be canceled. The region of oscillators that was adjacent to these, prior to the cut, is now on the section of the glass that is to the left. When the two sections were together, these scatterers presumably also emitted wavelets in the backward direction that were  $180^\circ$  out-of-phase with, and canceled, beam-I. Now they produce reflected beam-II. Each molecule scatters light in the backward direction, and, in principle, *each and every molecule contributes to the reflected wave*. Nonetheless, in practice, it is a thin layer ( $\approx \lambda/2$  deep) of unpaired atomic oscillators near the surface that is effectively responsible for the reflection. For an air–glass interface, about 4% of the energy of an incident beam falling perpendicularly *in air on glass* will be reflected straight back out by this layer of unpaired scatterers (p. 90). And that’s true whether the glass is 1.0 mm thick or 1.00 m thick.

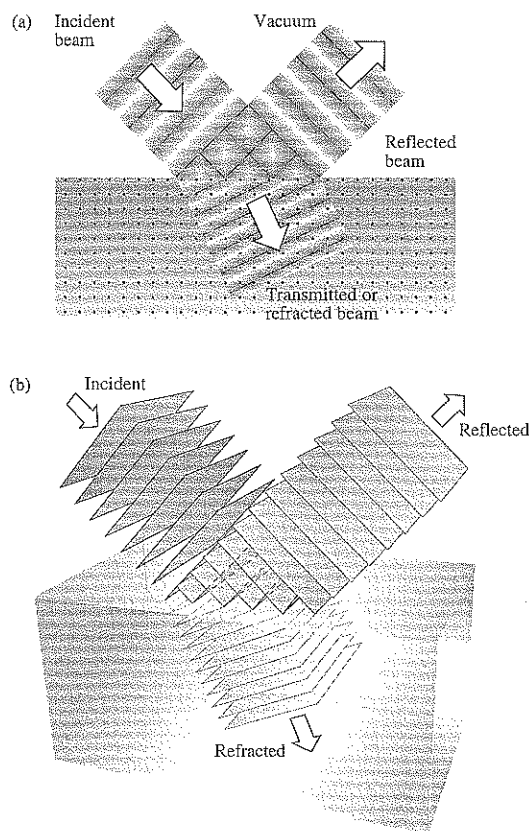
Beam-I reflects off the right-hand block, and because light was initially traveling from a less to a more optically dense medium, this is called **external reflection**. In other words, the index of the incident medium ( $n_i$ ) is less than the index of the transmitting medium ( $n_t$ ). Since the same thing happens to the unpaired layer on the section that was moved to the left, it, too, reflects backwards. With the beam incident perpendicularly *in glass on air*, 4% must again be reflected, this time as beam-II.



**Figure 4.12** (a) A lightbeam propagating through a dense homogeneous medium such as glass. (b) When the block of glass is cut and parted, the light is reflected backward at the two new interfaces. Beam-I is externally reflected, and beam-II is internally reflected. Ideally, when the two pieces are pressed back together, the two reflected beams cancel one another.

This process is referred to as **internal reflection** because  $n_i > n_t$ . If the two glass regions are made to approach one another increasingly closely (so that we can imagine the gap to be a thin film of, say, air—p. 402), the reflected light will diminish until it ultimately vanishes as the two faces merge and disappear and the block becomes continuous again. Remember this ***180° relative phase shift between internally and externally reflected light*** (see Section 4.10 for a more rigorous treatment)—we will come back to it later on.

Experience with the common mirror makes it obvious that white light is reflected as white—it certainly isn’t blue. To see why, first remember that the layer of scatterers responsible for the reflection is effectively about  $\lambda/2$  thick (as per Fig. 4.6). Thus the larger the wavelength, the deeper the region contributing (typically upward of a thousand atom layers), and the more scatterers there are acting together. This tends to balance out the fact that each scatterer is less efficient as  $\lambda$

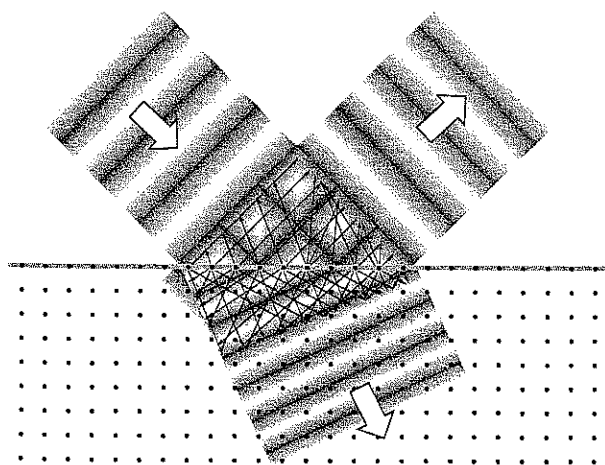


**Figure 4.13** A beam of plane waves incident on a distribution of molecules constituting a piece of clear glass or plastic. Part of the incident light is reflected and part refracted.

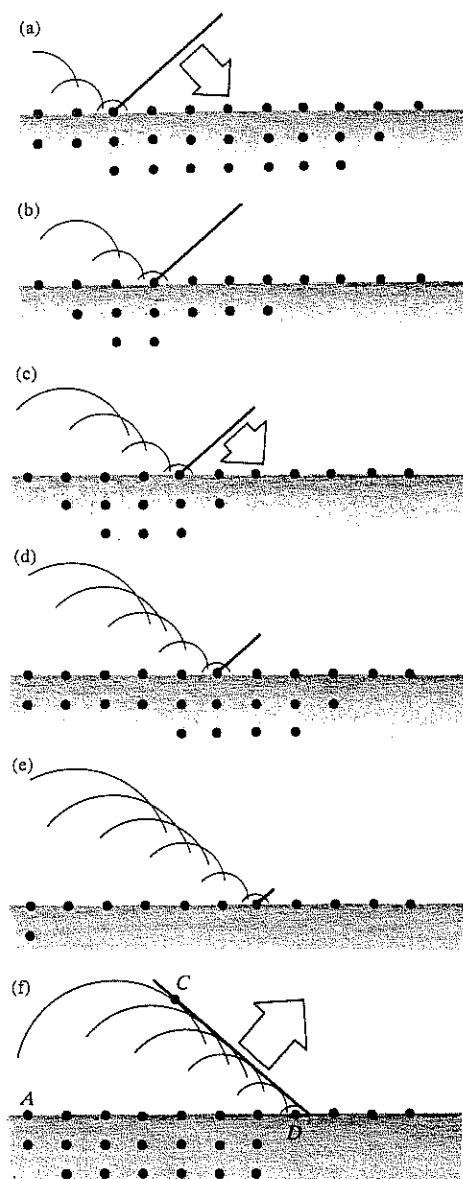
increases (remember  $1/\lambda^4$ ). The combined result is that *the surface of a transparent medium reflects all wavelengths about equally and doesn't appear colored in any way*. That, as we will see, is why this page looks white under white-light illumination.

### 4.3.1 The Law of Reflection

Figure 4.13 shows a beam composed of plane wavefronts impinging at some angle on the smooth, flat surface of an optically dense medium (let it be glass). Assume that the surrounding environment is vacuum. Follow one wavefront as it sweeps in and across the molecules on the surface. For the sake of simplicity, in Figs. 4.14 and 4.15 we have omitted everything but one molecular layer at the interface. As the wavefront descends, it energizes and re-energizes one scatterer after another, each of which radiates a stream of photons that can be regarded as a hemispherical wavelet in the incident medium. Because the wavelength is so much greater than the separation between the molecules, the wavelets emitted back into the incident medium advance together and add constructively in only one direction, and there is one well-defined *reflected beam*. That would not be true if the incident radiation was short-wavelength X-rays, in which circumstance there would be several reflected beams. And it would not be true if the scatterers were far apart compared to  $\lambda$ , as they are for a diffraction grating (p. 476), in which case there would also be several reflected beams. The direction of the reflected beam is



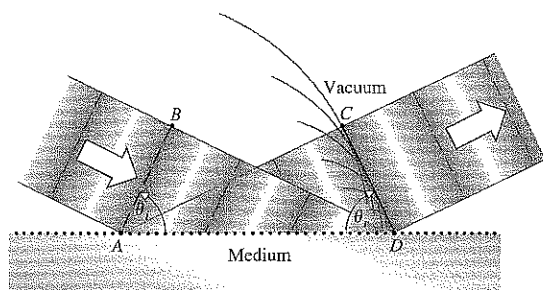
**Figure 4.14** A plane wave sweeps in stimulating atoms across the interface. These radiate and reradiate, thereby giving rise to both the reflected and transmitted waves.



**Figure 4.15** The reflection of a wave as the result of scattering.

determined by the constant phase difference between the atomic scatterers. That, in turn, is determined by the angle made by the incident wave and the surface, the so-called **angle-of-incidence**.

In Fig. 4.16, the line  $\overline{AB}$  lies along an incoming wavefront, while  $\overline{CD}$  lies on an outgoing wavefront—in effect,  $\overline{AB}$  transforms on reflection into  $\overline{CD}$ . With Fig. 4.15 in mind, we see



**Figure 4.16** Plane waves enter from the left and are reflected off to the right. The reflected wavefront  $\overline{CD}$  is formed of waves scattered by the atoms on the surface from A to D. Just as the first wavelet arrives at C from A, the atom at D emits, and the wavefront along  $\overline{CD}$  is completed.

that the wavelet emitted from A will arrive at C in-phase with the wavelet just being emitted from D (as it is stimulated by B), as long as the distances  $\overline{AC}$  and  $\overline{BD}$  are equal. In other words, if all the wavelets emitted from all the surface scatterers are to overlap in-phase and form a single reflected plane wave, it must be that  $\overline{AC} = \overline{BD}$ . Then, since the two triangles have a common hypotenuse

$$\frac{\sin \theta_i}{BD} = \frac{\sin \theta_r}{AC}$$

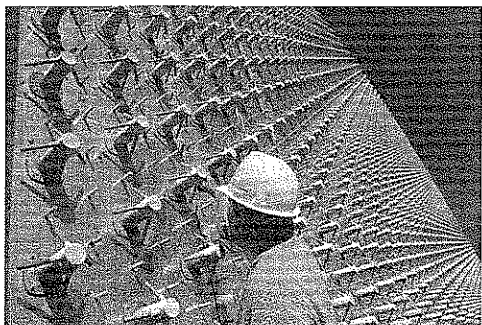
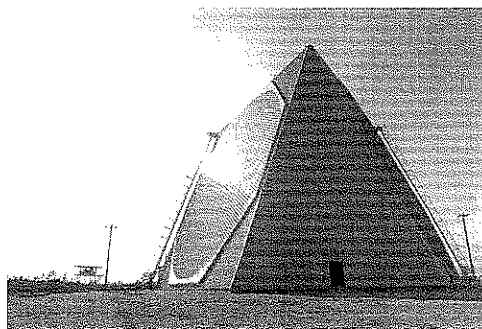
All the waves travel in the incident medium with the same speed  $v_i$ . It follows that in the time  $(\Delta t)$  it takes for point B on the wavefront to reach point D on the surface, the wavelet emitted from A reaches point C. In other words,  $\overline{BD} = v_i \Delta t = \overline{AC}$ , and so from the above equation,  $\sin \theta_i = \sin \theta_r$ , which means that

$$\theta_i = \theta_r \quad (4.3)$$

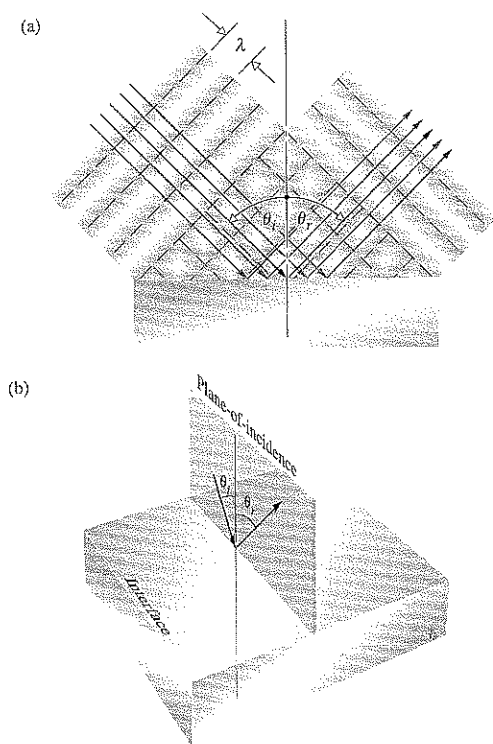
**The angle-of-incidence equals the angle-of-reflection.** This equation is the first part of the **Law of Reflection**. It initially appeared in the book *Catoptrics*, which was purported to have been written by Euclid. We say that a beam is *normally incident* when  $\theta_i = 0^\circ$ , in which case  $\theta_r = 0^\circ$  and for a mirror the beam reflects back on itself. Similarly, *glancing incidence* corresponds to  $\theta_i \approx 90^\circ$  and therefore  $\theta_r \approx 90^\circ$ .

## Rays

Drawing wavefronts can get things a bit cluttered, so we introduce another convenient scheme for visualizing the progression of light. The imagery of antiquity was in terms of straight-line streams of light, a notion that got into Latin as “radii” and reached English as “rays.” **A ray is a line drawn in space corresponding to the direction of flow of radiant energy.** It is a mathematical construct and not a physical entity. In a medium that is uniform (homogeneous), rays are straight. If the medium behaves in the same manner in every direction (isotropic), **the rays are perpendicular to the wavefronts.** Thus for a point source emitting spherical waves, the rays, which are perpendicular to them, point radially outward from the source. Similarly, the rays associated with plane waves are all parallel. Rather than sketching bundles of rays, we can simply draw one incident ray and one reflected ray (Fig. 4.17a). *All the angles are now measured from the perpendicular (or*



A modern phased-array radar system. The field of individual small antennas behaves very much like the atoms on a smooth surface. By introducing a proper phase shift between adjacent rows the antenna can “look” in any direction. A reflecting surface has a similar phase shift determined by  $\theta_i$  as the incident wave sweeps over the array of atoms. (Courtesy Raytheon Corp.)

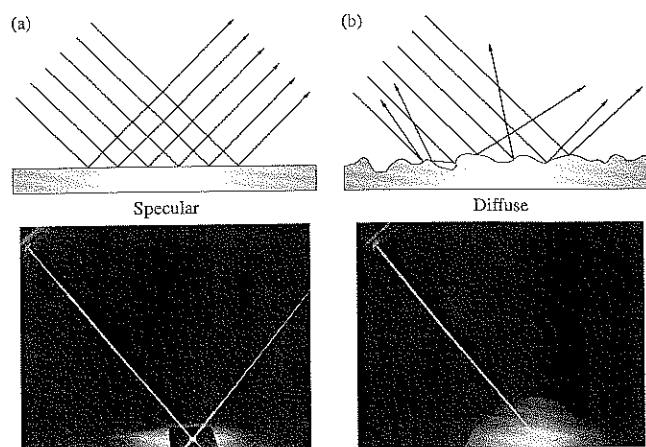


**Figure 4.17** (a) Select one ray to represent the beam of plane waves. Both the angle-of-incidence  $\theta_i$  and the angle-of-reflection  $\theta_r$  are measured from a perpendicular drawn to the reflecting surface. (b) The incident ray and the reflected ray define the *plane-of-incidence*, perpendicular to the reflecting surface.

normal) to the surface, and  $\theta_i$  and  $\theta_r$  have the same numerical values as before (Fig. 4.16).

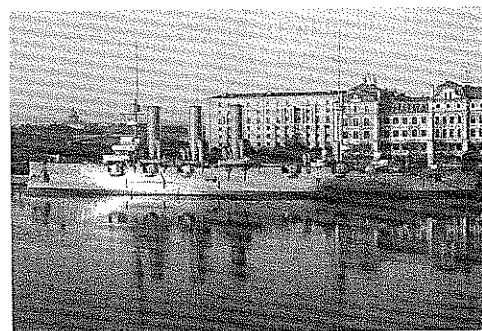
The ancient Greeks knew the Law of Reflection. It can be deduced by observing the behavior of a flat mirror, and nowadays that observation can be done most simply with a flashlight or, even better, a low-power laser. The second part of the Law of Reflection maintains that *the incident ray, the perpendicular to the surface, and the reflected ray all lie in a plane* called the **plane-of-incidence** (Fig. 4.17b)—this is a three-dimensional business. Try to hit some target in a room with a flashlight beam by reflecting it off a stationary mirror, and the importance of this second part of the law becomes obvious!

Figure 4.18a shows a beam of light incident upon a reflecting surface that is smooth (one for which any irregularities are small compared to a wavelength). In that case, the light re-

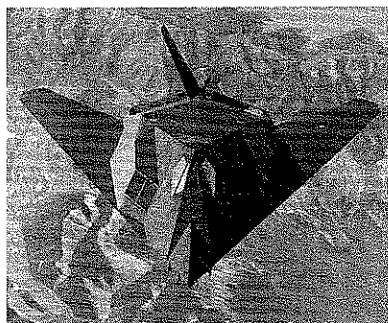


**Figure 4.18** (a) Specular reflection. (b) Diffuse reflection. (Photos courtesy Donald Dunitz.)

emitted by millions upon millions of atoms will combine to form a single well-defined beam in a process called **specular reflection** (from the word for a common mirror alloy in ancient times, *speculum*). Provided the ridges and valleys are small compared to  $\lambda$ , the scattered wavelets will still arrive more or less in-phase when  $\theta_i = \theta_r$ . This is the situation assumed in Figs. 4.13, 4.15, 4.16, and 4.17. On the other hand, when the surface is rough in comparison to  $\lambda$ , although the angle-of-incidence will equal the angle-of-reflection for each ray, the whole lot of rays will emerge every which way, constituting what is called **diffuse reflection** (see photo). Both of these conditions are extremes; the reflecting behavior of most



The cruiser *Aurora*, which played a key role in the Communist Revolution (1917), docked in St. Petersburg. Where the water is still, the reflection is specular. The image blurs where the water is rough and the reflection diffuse.



The F-117A Stealth fighter has an extremely small radar profile, that is, it returns very little of the incoming microwaves back to the station that sent them. That's accomplished mostly by constructing the aircraft with flat tilted-planes that use the Law of Reflection to scatter the radar waves away from their source. One wants to avoid  $\theta_i = \theta_r \approx 0$ .

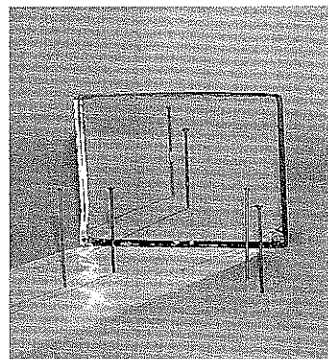
surfaces lies somewhere between them. Thus, although the paper of this page was deliberately manufactured to be a fairly diffuse scatterer, the cover of the book reflects in a manner that is somewhere between diffuse and specular.

## 4.4 Refraction

Figure 4.13 shows a beam of light impinging on an interface at some angle ( $\theta_i \neq 0$ ). The interface corresponds to a major inhomogeneity, and the atoms that compose it scatter light both backward, as the reflected beam, and forward, as the transmitted beam. The fact that the incident rays are bent or "turned out of their way," as Newton put it, is called **refraction**.

Examine the transmitted or refracted beam. Speaking classically, each energized molecule on the interface radiates wavelets into the glass that expand out at speed  $c$ . These can be imagined as combining into a secondary wave that then recombines with the unscattered remainder of the primary wave, to form the net transmitted wave. The process continues over and over again as the wave advances in the transmitting medium.

However we visualize it, immediately on entering the transmitting medium, there is a single net field, a single net wave. As we have seen, this transmitted wave usually propagates with an effective speed  $v_t < c$ . It's essentially as if the atoms at the interface scattered "slow wavelets" into the glass that combine to form the "slow transmitted wave." We'll come back to this imagery when we talk about Huygens's Principle. In any event, because the cooperative phenomenon

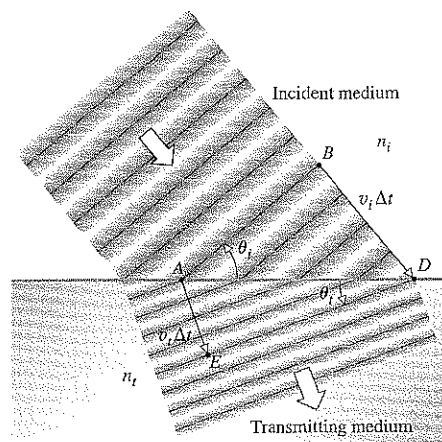


By placing a pair of pins in front of a flat mirror and aligning their images with another pair of pins, you can easily verify that  $\theta_i = \theta_r$ . (Photo by E.H.)

known as the transmitted electromagnetic wave *is* slower than the incident electromagnetic wave, the transmitted wavefronts are refracted, displaced (turned with respect to the incident wavefronts), and the beam bends.

### 4.4.1 The Law of Refraction

Figure 4.19 picks up where we left off with Figs. 4.13 and 4.16. The diagram depicts several wavefronts, all shown at a single instant in time. Remember that each wavefront is a surface of constant phase, and, to the degree that the phase of the



**Figure 4.19** The refraction of waves. The atoms in the region of the surface of the transmitting medium reradiate wavelets that combine constructively to form a refracted beam.



net field is retarded by the transmitting medium, each wavefront is held back, as it were. The wavefronts “bend” as they cross the boundary because of the speed change. Alternatively, we can envision Fig. 4.19 as a multiple-exposure picture of a single wavefront showing it after successive equal intervals of time. Notice that in the time  $\Delta t$ , which it takes for point  $B$  on a wavefront (traveling at speed  $v_i$ ) to reach point  $D$ , the transmitted portion of that same wavefront (traveling at speed  $v_t$ ) has reached point  $E$ . If the glass ( $n_t = 1.5$ ) is immersed in an incident medium that is vacuum ( $n_i = 1$ ) or air ( $n_i = 1.0003$ ) or anything else where  $n_t > n_i$ ,  $v_t < v_i$  and  $\overline{AE} < \overline{BD}$ , the wavefront bends. The refracted wavefront extends from  $E$  to  $D$ , making an angle with the interface of  $\theta_t$ . As before, the two triangles  $ABD$  and  $AED$  in Fig. 4.19 share a common hypotenuse ( $AD$ ), and so

$$\frac{\sin \theta_i}{\overline{BD}} = \frac{\sin \theta_t}{\overline{AE}}$$

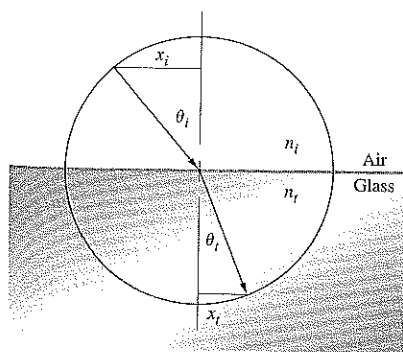
where  $\overline{BD} = v_i \Delta t$  and  $\overline{AE} = v_t \Delta t$ . Hence

$$\frac{\sin \theta_i}{v_i} = \frac{\sin \theta_t}{v_t}$$

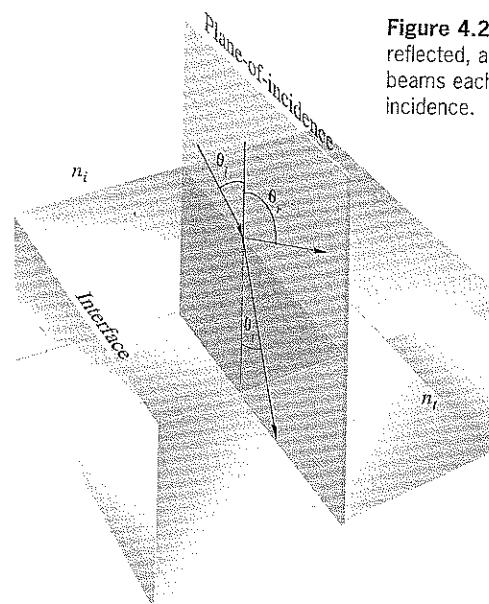
Multiply both sides by  $c$ , and since  $n_i = c/v_i$  and  $n_t = c/v_t$

$$n_i \sin \theta_i = n_t \sin \theta_t \quad (4.4)$$

This equation is the first portion of the **Law of Refraction**, also known as **Snell's Law** after the man who proposed it (1621), Willebrord Snel van Royen (1591–1626). Snel's analysis has been lost, but contemporary accounts follow the treatment shown in Fig. 4.20. What was found through observation was that the bending of the rays could be quantified via



**Figure 4.20**  
Descartes's arrangement for deriving the Law of Refraction.



**Figure 4.21** The incident, reflected, and transmitted beams each lie in the plane-of-incidence.

the ratio of  $x_i$  to  $x_t$ , which was constant for all  $\theta_i$ . That constant was naturally enough called the *index of refraction*. In other words,

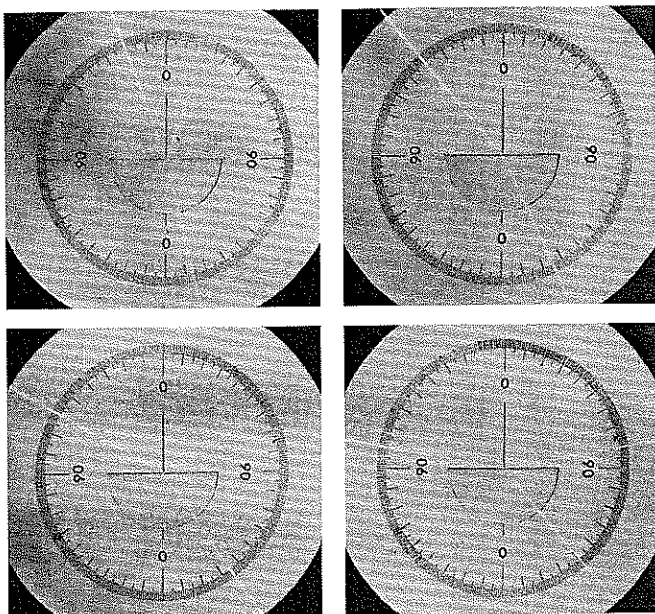
$$\frac{x_i}{x_t} \equiv n_t$$

and in air that's equivalent to Eq. (4.4). We now know that the Englishman Thomas Harriot had come to the same conclusion before 1601, but he kept it to himself.

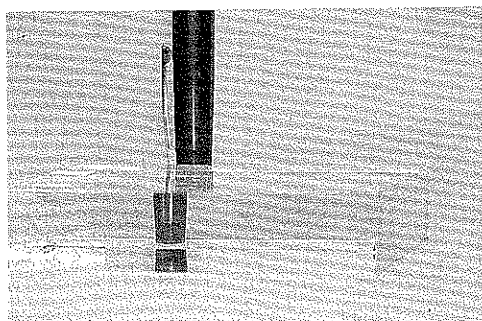
At first, the indices of refraction were simply experimentally determined constants of the physical media. Later, Newton was actually able to derive Snell's Law using his own corpuscular theory. By then, the significance of  $n$  as a measure of the speed of light was evident. Still later, Snell's Law was shown to be a natural consequence of Maxwell's Electromagnetic Theory (p. 112).

It is again convenient to transform the diagram into a ray representation (Fig. 4.21) wherein all the angles are measured from the perpendicular. Along with Eq. (4.4), there goes the understanding that *the incident, reflected, and refracted rays all lie in the plane-of-incidence*. In other words, the respective unit propagation vectors  $\hat{k}_i$ ,  $\hat{k}_r$ , and  $\hat{k}_t$  are coplanar (Fig. 4.22).

When  $n_i < n_t$  (that is, when the light is initially traveling within the lower-index medium), it follows from Snell's Law that  $\sin \theta_i > \sin \theta_t$ , and since the same function is everywhere positive between  $0^\circ$  and  $90^\circ$ , then  $\theta_i > \theta_t$ . Rather than going



**Figure 4.22** Refraction at various angles of incidence. Notice that the bottom surface is cut circular so that the transmitted beam within the glass always lies along a radius and is normal to the lower surface in every case. (Photos courtesy PSSC College Physics, D. C. Heath & Co., 1968.)



The image of a pen seen through a thick block of clear plastic. The displacement of the image arises from the refraction of light toward the normal at the air-plastic interface. If this arrangement is set up with a narrow object (e.g., an illuminated slit) and the angles are carefully measured, you can confirm Snell's Law directly. (Photo by E.H.)

straight through, *the ray entering a higher-index medium bends toward the normal* (Fig. 4.23a). The reverse is also true (Fig. 4.23b); that is, *on entering a medium having a lower index, the ray, rather than going straight through, will bend away from the normal* (see photo). Notice that this implies that the rays will traverse the same path going either way, into or out of either medium. The arrows can be reversed and the resulting picture is still true.

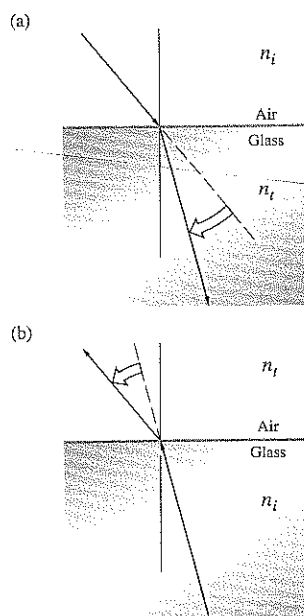
Snell's Law can be rewritten in the form

$$\frac{\sin \theta_i}{\sin \theta_t} = n_{ti} \quad (4.5)$$

where  $n_{ti} \equiv n_t/n_i$  is the **relative index of refraction** of the two media.

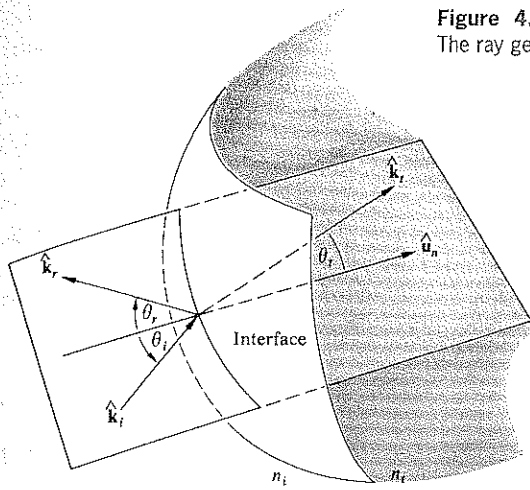
Let  $\hat{\mathbf{u}}_n$  be a unit vector normal to the interface pointing in the direction from the incident to the transmitting medium (Fig. 4.24). As you will have the opportunity to prove in Problem 4.29, the complete statement of the Law of Refraction can be written vectorially as

$$n_i(\hat{\mathbf{k}}_i \times \hat{\mathbf{u}}_n) = n_t(\hat{\mathbf{k}}_t \times \hat{\mathbf{u}}_n) \quad (4.6)$$



**Figure 4.23** The bending of rays at an interface. (a) When a beam of light enters a more optically dense medium, one with a greater index of refraction ( $n_i < n_t$ ), it bends toward the perpendicular. (b) When a beam goes from a more dense to a less dense medium ( $n_i > n_t$ ), it bends away from the perpendicular.

**Figure 4.24**  
The ray geometry.



or alternatively,

$$n_t \hat{k}_t - n_i \hat{k}_i = (n_t \cos \theta_t - n_i \cos \theta_i) \hat{u}_n \quad (4.7)$$

Fig. 4.19 illustrates the three important changes that occur in the beam traversing the interface. (1) It changes direction. Because the leading portion of the wavefront in the glass slows down, the part still in the air advances more rapidly, sweeping past and bending the wave toward the normal. (2) The beam in the glass has a broader cross section than the beam in the air; hence, the transmitted energy is spread thinner. (3) The wavelength decreases because the frequency is unchanged while the speed decreases;  $\lambda = v/\nu = c/n\nu$  and

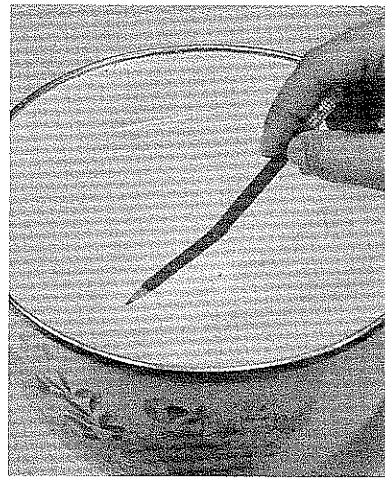
$$\lambda = \frac{\lambda_0}{n} \quad (4.8)$$

This latter notion suggests that the color aspect of light is better thought of as associated with its frequency (or energy,  $\mathcal{E} = h\nu$ ) than its wavelength, since the wavelength changes with the medium through which the light moves. Color is so much a physio-psychological phenomenon (p. 131) that it must be treated rather gingerly. Still, even though it's a bit simplistic, it's useful to remember that blue photons are more energetic than red photons. When we talk about wavelengths and colors, we should always be referring to **vacuum wavelengths** (henceforth to be given as  $\lambda_0$ ).

In all the situations treated thus far, it was assumed that the reflected and refracted beams always had the same frequency

as the incident beam, and that's ordinarily a reasonable assumption. Light of frequency  $\nu$  impinges on a medium and presumably drives the molecules into simple harmonic motion. That's certainly the case when the amplitude of the vibration is fairly small, as it is when the electric field driving the molecules is small. The  $E$ -field for bright sunlight is only about 1000 V/m (while the  $B$ -field is less than a tenth of the Earth's surface field). This isn't very large compared to the fields keeping a crystal together, which are of the order of  $10^{11}$  V/m—just about the same magnitude as the cohesive field holding the electron in an atom. We can usually expect the oscillators to vibrate in simple harmonic motion, and so the frequency will remain constant—the medium will ordinarily respond linearly. That will not be true, however, if the incident beam has an exceedingly large-amplitude  $E$ -field, as can be the case with a high-power laser. So driven, at some frequency  $\nu$  the medium can behave in a nonlinear fashion, resulting in reflection and refraction of harmonics ( $2\nu$ ,  $3\nu$ , etc.) in addition to  $\nu$ . Nowadays, second-harmonic generators (p. 641) are available commercially. You shine red light (694.3 nm) into an appropriately oriented transparent nonlinear crystal (of, for example, potassium dihydrogen phosphate, KDP, or ammonium dihydrogen phosphate, ADP) and out will come a beam of UV (347.15 nm).

One feature of the above treatment merits some further discussion. It was reasonably assumed that each point on the interface in Fig. 4.13 coincides with a particular point on each of the incident, reflected, and transmitted waves. In other words, there is a fixed phase relationship between each of the



Rays from the submerged portion of the pencil bend on leaving the water as they rise toward the viewer. (Photo by E.H.)

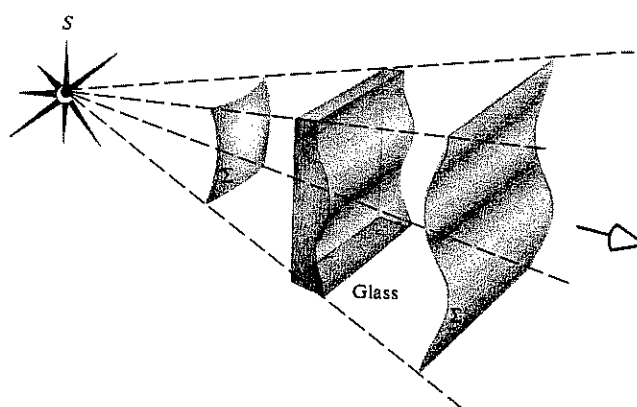
waves at all points along the interface. As the incident front sweeps across the interface, every point on it in contact with the interface is also a point on both a corresponding reflected front and a corresponding transmitted front. This situation is known as **wavefront continuity**, and it will be justified in a more mathematically rigorous treatment in Section 4.6.1. Interestingly, Sommerfeld\* has shown that the laws of reflection and refraction (independent of the kind of wave involved) can be derived directly from the requirement of wavefront continuity and the solution to Problem 4.26 demonstrates as much.

#### 4.4.2 Huygens's Principle

Suppose that light passes through a nonuniform sheet of glass, as in Fig. 4.25, so that the wavefront  $\Sigma$  is distorted. How can we determine its new form  $\Sigma'$ ? Or for that matter, what will  $\Sigma'$  look like at some later time, if it is allowed to continue unobstructed?

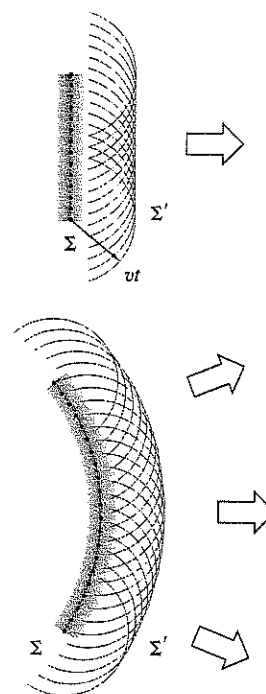
A preliminary step toward the solution of this problem appeared in print in 1690 in the work entitled *Traité de la Lumière*, which had been written 12 years earlier by the Dutch physicist Christiaan Huygens. It was there that he enunciated what has since become known as **Huygens's Principle**: *every point on a propagating wavefront serves as the source of spherical secondary wavelets, such that the wavefront at some later time is the envelope of these wavelets.*

A further crucial point is that *if the propagating wave has a frequency  $\nu$ , and is transmitted through the medium at a speed  $v$ , then the secondary wavelets have that same frequency and speed.* Huygens was a brilliant scientist, and this is the basis of a remarkably insightful, though quite naive, scattering theory. It's a very early treatment and naturally has several shortcomings, one of which is that it doesn't overtly incorporate the concept of interference and perforce cannot deal with lateral scattering. Moreover, the idea that the secondary wavelets propagate at a speed determined by the medium (a speed that may even be anisotropic, e.g., p. 340) is a happy guess. Nonetheless, Huygens's Principle can be used to arrive at Snell's Law in a way that's similar to the treatment that led to Eq. (4.4). It's probably best not to fuss over the physical details (such as how to rationalize propagation in vac-



**Figure 4.25** Distortion of a portion of a wavefront on passing through a material of nonuniform thickness.

uum) and just use the Principle as a tool—a highly useful fiction that works. After all, if Einstein is right, there are only scattered photons; the wavelets themselves are a theoretical construct.



**Figure 4.26** According to Huygens's Principle, a wave propagates as if the wavefront were composed of an array of point sources, each emitting a spherical wave.

\*A. Sommerfeld, *Optics*, p. 151. See also J. J. Sein, *Am. J. Phys.* **50**, 180 (1982).