Introduction to Jones Vectors

In 1941 R. Clark Jones developed a method to represent polarization of light using vectors. Named after its inventor, these vector are called **Jones Vectors.** A Jones Vector has the form

$$|\psi\rangle = \begin{pmatrix} a_x e^{i\phi_x} \\ a_y e^{i\phi_y} \end{pmatrix}$$

where a_x and a_y are the amplitudes of the x and y components of the polarization, and ϕ_x and ϕ_y are the phases of the x and y components. We will use the **right hand circularly** polarized light as an example to show how the Jones Vectors are used

For a **right hand circularly** polarized light, the polarization vector rotates in the clockwise direction. Therefore the x and y components of the polarization have the same magnitude but the phase of the y component is $90°$ or $\pi/2$ behind the x component, so its Jones Vector is

$$|R\rangle = \begin{pmatrix} a \\ ae^{i\pi/2} \end{pmatrix} = a\begin{pmatrix} 1 \\ i \end{pmatrix}$$

In many applications, we are not interested in the exact amplitudes. In these cases we would simplify the mathematics by normalizing the Jones Vector, i.e. the magnitude of the vector is $\langle\psi|\psi\rangle = 1$. Therefore with some simplification, the normalized Jones Vector of **right hand circularly** polarized light is

$$|R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

The following is a table to summarizes a few special cases of polarization and their normalized Jones Vectors,

Linearly polarized light in the x direction	$\|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
Linearly polarized light in the y direction	$\|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Right hand circularly polarized light	$\|R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$
Left hand circularly polarized light	$\|L\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Since Jones Vectors represents the state of polarization then the optical element that changes the light from one state to another is represented by a matrix, call a **Jones Matrix.** If a beam of light with Jones Vector $|\psi_1\rangle$ pass thought an optics with Jones Matrix T then the final polarization is represented by the Jones Vector $|\psi_2\rangle$, this process then can be represented by the following equation

$$|\psi_2\rangle = T|\psi_1\rangle$$

If the beam of light passes through a system of optical elements in the order, T_1, T_2,..., T_n, then the equation is

$$|\psi_2\rangle = T_n T_{n-1} ......T_1 |\psi_1\rangle$$

Notice that in the equation, the order of the matrices is reversed. In such a system, we can always represent the it by one Jones Matrix, T where

$$T = T_n T_{n-1} ......T_1$$

The following table shows the Jones Matrices for some of the optical elements.

Linear polarizer along the x - direction	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
Linear polarizer along the y - direction	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
Half - wave plate	$\begin{pmatrix} 1 & 0 \\ 0 & e^{-ip} \end{pmatrix}$