

Properties of the unilateral Laplace transform

| | Time domain | s domain |
|-------------------------------------|--|--|
| Linearity | $af(t) + bg(t)$ | $aF(s) + bG(s)$ |
| Frequency-domain derivative | $tf(t)$ | $-F'(s)$ |
| Frequency-domain general derivative | $t^n f(t)$ | $(-1)^n F^{(n)}(s)$ |
| Derivative | $f'(t)$ | $sF(s) - f(0)$ |
| Second derivative | $f''(t)$ | $s^2 F(s) - sf(0) - f'(0)$ |
| General derivative | $f^{(n)}(t)$ | $s^n F(s) - \sum_{k=1}^n s^{n-k} f^{(k-1)}(0)$ |
| Frequency-domain integration | $\frac{1}{t} f(t)$ | $\int_s^\infty F(\sigma) d\sigma$ |
| Time-domain integration | $\int_0^t f(\tau) d\tau = (u * f)(t)$ | $\frac{1}{s} F(s)$ |
| Frequency shifting | $e^{at} f(t)$ | $F(s - a)$ |
| Time shifting | $f(t - a)u(t - a)$ | $e^{-as} F(s)$ |
| Time scaling | $f(at)$ | $\frac{1}{a} F\left(\frac{s}{a}\right)$ |
| Multiplication | $f(t)g(t)$ | $\frac{1}{2\pi i} \lim_{T \rightarrow \infty} \int_{c-iT}^{c+iT} F(\sigma)G(s - \sigma) d\sigma$ |
| Convolution | $(f * g)(t) = \int_0^t f(\tau)g(t - \tau) d\tau$ | $F(s) \cdot G(s)$ |