CHAPTER 15

Coherent Interactions of a Radiation Field and an Atomic System

15.0 INTRODUCTION

In the treatment, up to this point, of the interaction of radiation and an atomic system, we considered only equilibrium situations. It was assumed that the duration of the interaction was long compared to the inelastic (τ) and elastic (T_2) collision times so that the atomic medium response can be described by means of the susceptibility, χ, as in (8.1-19).

In situations involving intense optical fields and/or long relaxation times, we are often concerned with the atomic response to the field on a time scale *shorter* than the collision times. In such cases, the atomic polarization is *not an explicit function of the instantaneous electric field.*

A number of new phenomena occur in this regime and their study requires some new analytic tools. Some of these phenomena are photon echoes, superradiant states, and self-induced transparency.

Before considering these phenomena, we introduce a formalism, due to Feynman, Vernon, and Hellwarth (Reference 1), which establishes a formal similarity between the response of a two-level atomic system to that of a magnetic spin in a dc magnetic field. This point of view makes it possible to visualize the atomic dipolar behavior in terms of the conceptually simpler spin precession. The "equations of motion" describing the evolution of the atomic system are shown to be identical to those of a gyromagnet in a dc magnetic field.

15.1 VECTOR REPRESENTATION OF THE INTERACTION OF A RADIATION FIELD WITH A TWO-LEVEL ATOMIC SYSTEM (REFERENCE 1)

The Schrödinger equation, when applied to the interaction of a two-level atomic system with an electromagnetic field, can be cast in a simple but rigorous, geometric form. The evolution of the atomic wavefunction can be represented by the motion of a fictitious vector. This formalism is especially useful in physical situations where collision processes can be ignored. This is the case, for example, in the propagation of intense ultrashort pulses. We will

use it to treat the problem of optical nutation, spin echoes, and self-induced transparency.

We need to solve the Schrödinger equation

$$\mathcal{H}\psi = i\hbar \frac{\partial \psi}{\partial t} \tag{15.1-1}$$

where

$$\mathcal{H} = \mathcal{H}_0 + V(t) \tag{15.1-2}$$

$V(t)$ is the Hamiltonian representing the interaction of the atomic system with the electromagnetic field. $\mathcal{H}_0$ is the Hamiltonian with zero field.

The wavefunction of an individual system in an ensemble of noninteracting systems can be taken in the form

$$\psi(t) = a(t)u_a + b(t)u_b \tag{15.1-3}$$

where u_a and u_b are the time-independent eigenfunctions of $\mathcal{H}_0$,

$$\mathcal{H}_0 u_a = \frac{\hbar\omega}{2} u_a$$

$$\mathcal{H}_0 u_b = -\frac{\hbar\omega}{2} u_b$$

and the interaction is assumed to involve transitions between states $|a\rangle$ and $|b\rangle$ only, whose energy separation is $\hbar\omega$ (see Figure 15.1).

In general, we need four constants to completely specify $\psi(t)$. These are the real and imaginary parts of $a(t)$ and $b(t)$. Since the absolute phase of $\psi(t)$ has no physical significance, we need only three. These can be taken as the magnitudes of $a(t)$ and $b(t)$ and their relative phase. Alternatively, we can construct three real functions (r_1, r_2, r_3) of a and b that can be viewed as the components of a vector $\mathbf{r}$ in some mathematical space with coordinate systems labeled (1, 2, 3).

$$\begin{aligned} r_1 &= ab^* + ba^* \\ r_2 &= i(ab^* - ba^*) \\ r_3 &= aa^* - bb^* \end{aligned} \tag{15.1-4}$$

so that $|\mathbf{r}|^2 = (|a|^2 + |b|^2)^2 = (\int \psi^*\psi \, dv)^2 = 1$.

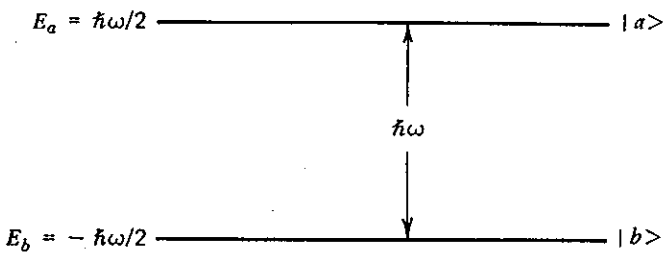

FIGURE 15.1 The two level atomic system used in the vector model (Section 15.1).

In terms of the density matrix defined in Section 8.1, we have

$$\begin{aligned} r_1 &= 2\,\mathrm{Re}(\rho_{21}) \\ r_2 &= -2\,\mathrm{Im}(\rho_{21}) \\ r_3 &= \rho_{22} - \rho_{11} \end{aligned} \tag{15.1-5}$$

The time dependence of $\mathbf{r}$ can be obtained from the Schrödinger equation (15.1-1) written as

$$(\mathcal{H}_0 + V)(au_a + bu_b) = i\hbar(\dot{a}u_a + \dot{b}u_b) \tag{15.1-6}$$

Multiplying (15.1-6) by u_a^* and integrating over all space lead to

$$\frac{da}{dt} = -\frac{i}{\hbar}\left[a\left(\frac{\hbar\omega}{2} + V_{aa}\right) + bV_{ab}\right] \tag{15.1-7a}$$

and repeating the same procedure with u_b^* gives

$$\frac{db}{dt} = -\frac{i}{\hbar}\left[b\left(-\frac{\hbar\omega}{2} + V_{bb}\right) + aV_{ba}\right] \tag{15.1-7b}$$

In the following, we will limit ourselves to situations where V_{aa}, $V_{bb} \ll \hbar\omega$[1]. If we neglect V_{aa} and V_{bb} in (15.1-7), we can show that the state vector $\mathbf{r}$ defined by (15.1-4) obeys the simple equation

$$\frac{d\mathbf{r}}{dt} = \boldsymbol{\omega}(t) \times \mathbf{r} \tag{15.1-8}$$

where

$$\begin{aligned} \omega_1 &\equiv (V_{ab} + V_{ba})/\hbar \\ \omega_2 &\equiv i(V_{ab} - V_{ba})/\hbar \\ \omega_3 &\equiv \omega \end{aligned} \tag{15.1-9}$$

As a proof, consider the 1 component of (15.1-8)

$$\frac{dr_1}{dt} = \omega_2 r_3 - \omega_3 r_2 \tag{15.1-10}$$

The left side of (15.1-10) is equal to $\dot{a}b^* + a(\dot{b})^* + \text{c.c.}$ If we use (15.1-7) with $V_{aa} = V_{bb} = 0$, it becomes

$$\frac{dr_1}{dt} = \frac{i}{\hbar}(-\hbar\omega ab^* + V_{ba}^* aa^* - V_{ab}bb^*) + \text{c.c.}$$

Using (15.1-4) and (15.1-9), we obtain

$$\omega_2 r_3 - \omega_3 r_2 = \frac{i}{\hbar}(V_{ab} - V_{ba})(aa^* - bb^*) - i\omega(ab^* - ba^*)$$

[1] Both V_{aa} and V_{bb} are zero in magnetic dipole transitions between the states $m_s = \pm\frac{1}{2}$ of a "spin $\frac{1}{2}$" system, or in electric dipole $\Delta m = \pm 1$ transitions, in a situation where $\mathcal{H}_0$ possesses inversion symmetry so that u_a and u_b have definite parity.

which is the same as the expression for dr_1/dt. The proof for the 2 and 3 components of (15.1-8) is similar.

To proceed further, we need to be more specific about the transition $a \rightarrow b$ and the electromagnetic field. Let us consider the important class of dipole transitions involving the selection rule $\Delta m = \pm 1$. Our notation will correspond to electric dipole transitions. The interaction Hamiltonian in this case becomes

$$V = -\mu_x E_x - \mu_y E_y \tag{15.1-11}$$

Defining

$$\begin{aligned} \mu^+ &\equiv \mu_x + i\mu_y \qquad E^+ \equiv E_x + iE_y \\ \mu^- &\equiv \mu_x - i\mu_y \qquad E^- \equiv E_x - iE_y \end{aligned} \tag{15.1-12}$$

gives

$$V = -\tfrac{1}{2}(\mu^+ E^- + \mu^- E^+) \tag{15.1-13}$$

For $\Delta m = \pm 1$ transitions, we have

$$\begin{aligned} \langle m+1|\ \mu^-\ |m\rangle &= 0 \\ \langle m|\ \mu^+\ |m+1\rangle &= 0 \end{aligned} \tag{15.1-14}$$

so that from (15.1-12) and (15.1-13) and letting $u_a \rightarrow |m+1\rangle$, $u_b \rightarrow |m\rangle$

$$\begin{aligned} V_{ab} &= -\tfrac{1}{2}\mu_{ab}^+(E_x - iE_y) \\ V_{ba} &= -\tfrac{1}{2}\mu_{ba}^-(E_x + iE_y) \end{aligned} \tag{15.1-15}$$

we are free to choose the phases of u_a and u_b so that μ_{ab}^+ is a real positive number that we designate as 2μ.

$$\mu_{ab}^+ = \mu_{ba}^- \equiv 2\mu \qquad \mu = \langle b|\ \mu_x\ |a\rangle \tag{15.1-16}$$

with the equality following from the fact that $\mu_{ab}^+ = (\mu_{ba}^-)^*$. From (15.1-9) and (15.1-15), we get

$$\begin{aligned} \omega_1(t) &= (V_{ab} + V_{ba})/\hbar = -\frac{2\mu E_x(t)}{\hbar} \\ \omega_2(t) &= i(V_{ab} - V_{ba})/\hbar = -\frac{2\mu E_y(t)}{\hbar} \end{aligned} \tag{15.1-17}$$

so that the vector $\boldsymbol{\omega}$ behaves in the mathematical 1–2 plane exactly as the vector $\mathbf{E}$ does in the physical x–y plane. To attach, similarly, a physical significance to $\mathbf{r}$, consider the expectation value of a transverse (e.g., x) component of the dipole operator,

$$\langle \mu_x \rangle = \tfrac{1}{2}\langle \mu^+ + \mu^- \rangle = \tfrac{1}{2}\int (a^* u_a^* + b^* u_b^*)(\mu^+ + \mu^-)(a u_a + b u_b)\, dv$$

that, using (15.1-14), gives

$$\langle \mu_x \rangle = \mu r_1 \qquad \text{and} \qquad \langle \mu_y \rangle = \mu r_2 \tag{15.1-18}$$

so that the expectation value of the dipole moment operator (which corresponds to the radiating dipole of one atomic system) behaves in the physical x–y plane in the same way as the $\mathbf{r}$ vector in the fictitious 1–2 plane.

The machinery we have just constructed for treating the problem of the dipole interaction of a two-level atomic system with an electromagnetic field is thus apparent. All we need to do is solve the vector equation

$$\frac{d\mathbf{r}}{dt} = \boldsymbol{\omega}(t) \times \mathbf{r} \tag{15.1-8}$$

for $\mathbf{r}(t)$ where $\boldsymbol{\omega}$ is given by (15.1-17). The transverse dipole moments of a single atomic system are given directly by $\mathbf{r}$ using (15.1-18). Since the wave function $\psi(t)$ is related uniquely to $\mathbf{r}$ via (15.1-4), a knowledge of $\mathbf{r}(t)$ is formally equivalent to a complete (in the quantum mechanical sense) specification of the system. The procedure outlined above requires a knowledge of the initial value $\mathbf{r}(0)$, which is equivalent to specifying $\psi(0)$ when solving the Schrödinger equation.

In the following sections, we will study the solution of (15.1-8) and its implications in a few simple cases. Before we do that, however, we may find it instructive to consider the significance of (15.1-8) in the simple case of a spin 1/2 magnetic system. Here, we have $u_a = |1/2\rangle$, $u_b = |-1/2\rangle$ and

$$\boldsymbol{\mu} = \frac{2\beta}{\hbar}\mathbf{S}$$

where β is the Bohr magneton, $\boldsymbol{\mu}$ the magnetic dipole moment, and $\mathbf{S}$ the spin angular momentum operator. Using (15.1-16), we get

$$2\mu \equiv \mu_{ab}^{+} = \frac{2\beta}{\hbar}\langle \tfrac{1}{2}|\, S^{+}\, |-\tfrac{1}{2}\rangle = 2\beta$$

From (15.1-18), it follows that

$$\langle \mu_x \rangle = \beta r_1 \qquad \langle \mu_y \rangle = \beta r_2$$

In this case, we also have

$$\langle \mu_z \rangle = \langle a u_{1/2} + b u_{-1/2}| \frac{2\beta}{\hbar} S_z |a u_{1/2} + b u_{-1/2}\rangle$$

$$= \beta(aa^* - bb^*) = \beta r_3$$

so that the identification of $\langle \boldsymbol{\mu} \rangle$ with $\beta\mathbf{r}$ is complete.

Using (15.1-17) and replacing $E_{x,y}$ with the magnetic field component $H_{x,y}$ give

$$\omega_1 = -\frac{2\beta H_x}{\hbar} \qquad \omega_2 = -\frac{2\beta H_x}{\hbar}$$

All that remains is to show that $\omega_3 = -2\beta H_z/\hbar$. This is done by recognizing that the levels' energy separation $\hbar\omega$ is given in this case by $-2\beta H_z$ so that

$$\hbar\omega_3 = \hbar\omega = -2\beta H_z$$

There is, thus, also a complete correspondence between $\boldsymbol{\omega}(t)$ and $-2\beta\mathbf{H}(t)/\hbar$. Using this correspondence as well as $\langle\boldsymbol{\mu}\rangle \rightarrow \beta\mathbf{r}$, derived above, in (15.1-8) gives

$$\frac{d\langle\boldsymbol{\mu}\rangle}{dt} = \gamma\langle\boldsymbol{\mu}\rangle \times \mathbf{H} \tag{15.1-19}$$

where $\gamma \equiv 2\beta/\hbar$ is the gyromagnetic ratio. This is the well-known equation of motion of a gyromagnet in a magnetic field.

This simple physical correspondence between r_3 and μ_z does not generally exist in the case of electric dipole transitions. As a matter of fact, in the electric $\Delta m = \pm 1$ case considered here, we have

$$\langle\mu_z\rangle \propto \langle au_a + bu_b|\, z\, |au_a + bu_b\rangle = 0$$

since both $\langle u_a|\, z\, |u_a\rangle$ and $\langle u_b|\, z\, |u_a\rangle = 0$, the first because of the odd parity of the integrand $|u_a|^2 z$ and the second because it involves the integral $\int_0^{2\pi} \exp(i\phi)\, d\phi$. The correspondence between the transverse components of $\mathbf{r}$ and $\boldsymbol{\mu}$ given by (15.1-18) is valid, however, and, as shown below, is extremely useful. The component r_3, in this case, is proportional to the expectation value of the unperturbed Hamiltonian since

$$\begin{aligned}\langle H_0\rangle &= \int \psi^* H_0 \psi\, dv = (aa^* - bb^*)\frac{\hbar\omega}{2} \\ &= r_3\frac{\hbar\omega}{2}\end{aligned} \tag{15.1-20}$$

Transformation to a Rotating Coordinate System

The solution of the dipolar equation of motion (15.1-8)

$$\frac{d\mathbf{r}}{dt} = \boldsymbol{\omega} \times \mathbf{r}$$

is greatly facilitated by transforming from the stationary (1, 2, 3) coordinate system to one that is rotating about it at a (radian) rate $\boldsymbol{\Omega}$. According to a basic theorem in vector calculus, the rate of change $d\mathbf{r}_R/dt$ of any vector $\mathbf{r}$ as *observed in the rotating system* is related to that observed in the stationary one by

$$\frac{d\mathbf{r}_R}{dt} = \left(\frac{d\mathbf{r}}{dt}\right)_R - \boldsymbol{\Omega} \times \mathbf{r}_R \tag{15.1-21}$$

where the R subscript indicates vector transformation to the rotated system.[2] Applying (15.1-21) to (15.1-8) leads to

$$\frac{d\mathbf{r}_R}{dt} = (\boldsymbol{\omega}_R - \boldsymbol{\Omega}) \times \mathbf{r}_R \tag{15.1-22}$$

[2] If a vector $\mathbf{A} = (A_1, A_2, A_3)$ in the (1, 2, 3) system, then in a system (I, II, III) rotated by an angle Ωt about the "3" axis, it becomes $\mathbf{A}_R = (A_\mathrm{I}, A_\mathrm{II}, A_\mathrm{III})$, where $A_\mathrm{I} = A_1 \cos\Omega t + A_2 \sin\Omega t$, $A_\mathrm{II} = -A_1 \sin\Omega t + A_2 \cos\Omega t$, $A_\mathrm{III} = A_3$. See Problem 15.4 for a related discussion.

The Behavior of $\mathbf{r}(t)$ with No Applied Field

Consider the solution of $\mathbf{r}(t)$ with no radiation field present. In this case, we obtain, from (15.1-17),

$$\omega_1 = 0 \qquad \omega_2 = 0 \qquad \omega_3 = \omega \Rightarrow \boldsymbol{\omega} = \mathbf{a}_3\omega$$

where $\mathbf{a}_3$ is a unit vector along the 3 direction. It is convenient to choose the rotation axis to be parallel to $\mathbf{a}_3$, in this case, designating the axes in the rotating system as I, II, III, we have $\boldsymbol{\Omega} = \mathbf{a}_{\text{III}}\Omega$ and (15.1-22) becomes

$$\frac{d\mathbf{r}_R}{dt} = \mathbf{a}_{\text{III}}(\omega - \Omega) \times \mathbf{r}_R \tag{15.1-23}$$

It may be useful to recall at this point that the choice of the direction and magnitude of $\boldsymbol{\Omega}$ is strictly a matter of convenience, so that if we take $\Omega = \omega$, the solution of (15.1-23) is

$$\mathbf{r}_R = \text{const}$$

and transforming back to the (1, 2, 3) system, we have from Figure 15.2 (for $\Omega = \omega$)

$$\begin{aligned} r_1 &= r_{\text{I}} \cos \omega t - r_{\text{II}} \sin \omega t \\ r_2 &= r_{\text{I}} \sin \omega t + r_{\text{II}} \cos \omega t \\ r_3 &= \sqrt{1 - r_{\text{I}}^2 - r_{\text{II}}^2} = \text{const} \end{aligned} \tag{15.1-24}$$

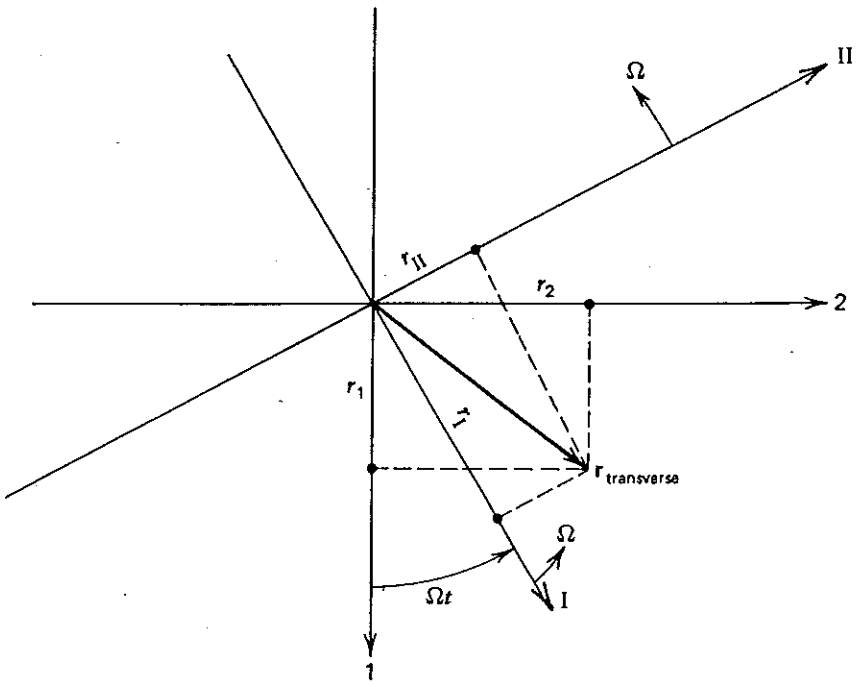

FIGURE 15.2 The relationship between the transverse components of $\mathbf{r}$ in the (1,2) and the rotating (I, II) coordinate system. System I, II, III rotates at a radian rate Ω about the 3 axis (axes 3 and III coincide).

The motion of $\mathbf{r}(t)$ is thus one of precession at a rate ω about the 3 axis at an arbitrary inclination angle so that r_3 and, according to (15.1-20), the energy are constants of the motion. Since $r_3 = (aa^* - bb^*)$, its constancy is just a geometrical representation of the fact that with no applied field, no transitions between $|a\rangle$ and $|b\rangle$ can take place, so that $|a|^2$ and $|b|^2$ are constant.[3]

The components r_{I} and r_{II} can be related simply to the amplitudes U and V of the medium polarization

$$P_x = U \cos \omega t - V \sin \omega t$$

From (15.1-18) and (15.1-24), we obtain

$$P_x = N\langle \mu_x \rangle = N\mu r_{\mathrm{I}} = N\mu(r_{\mathrm{I}} \cos \omega t - r_{\mathrm{II}} \sin \omega t)$$

so that

$$U = N\mu r_{\mathrm{I}} \qquad V = N\mu r_{\mathrm{II}}$$

where N is the density of atoms. If at $t = 0$ there are N_a atoms/m^3 in the upper level and N_b in the lower one then

$$U = (N_a - N_b)\mu r_{\mathrm{I}} \qquad V = (N_a - N_b)\mu r_{\mathrm{II}}$$

where $\mathbf{r}_{\mathrm{I,II}}(t)$ is the solution corresponding to an atom that is initially in the upper level $|a\rangle$, that is, $r_{\mathrm{III}}(0) = 1$. The proof of this last statement is left as a problem.

The Behavior of $\mathbf{r}(t)$ in an Applied Field

Consider next the behavior of $\mathbf{r}(t)$ when a circularly polarized electric field

$$\begin{aligned} E_x &= E \cos \omega_0 t \\ E_y &= E \sin \omega_0 t \end{aligned} \tag{15.1-25}$$

is applied in the x, y plane. From (15.1-17), the components of $\boldsymbol{\omega}(t)$ are

$$\begin{aligned} \omega_1 &= -\frac{2\mu}{\hbar} E \cos \omega_0 t \\ \omega_2 &= -\frac{2\mu}{\hbar} E \sin \omega_0 t \\ \omega_3 &= \omega \end{aligned} \tag{15.1-26}$$

so that, viewed in the 1, 2 plane, $\boldsymbol{\omega}(t)$ is a circularly polarized vector rotating at a rate ω_0 about the 3 axis with a constant magnitude $-2\mu E/\hbar$. In a coordinate system rotating in synchronism with $\boldsymbol{\omega}(t)$ (i.e., $\boldsymbol{\Omega} = \mathbf{a}_{\mathrm{III}}\omega_0$), the vector $\boldsymbol{\omega}(t)$ becomes stationary so that $\boldsymbol{\omega}_R = (-2\mu E/\hbar, 0, \omega)$ and the equation of

[3] The possibility of spontaneous transition from $|a\rangle$ to $|b\rangle$ is not included in the model due to the classical representation of the electric field in the Hamiltonian (15.1-11).

motion (15.1-22) becomes

$$\frac{d\mathbf{r}_R(\omega)}{dt} = \left[\mathbf{a}_{\mathrm{I}}\left(-\frac{2\mu E}{\hbar}\right) + \mathbf{a}_{\mathrm{III}}(\omega - \omega_0)\right] \times \mathbf{r}_R(\omega) \equiv \boldsymbol{\omega}_{\mathrm{eff}} \times \mathbf{r}_R(\omega) \quad (15.1\text{-}27)$$

The problem has thus been reduced to that of the precession of $\mathbf{r}_R$ about a *stationary* vector $\boldsymbol{\omega}_{\mathrm{eff}} = \mathbf{a}_{\mathrm{I}}[-(2\mu E/\hbar)] + \mathbf{a}_{\mathrm{III}}(\omega - \omega_0)$. If we use the solution (15.1-24) for a stationary $\boldsymbol{\omega}$, the rate of precession is

$$\omega_e = |\boldsymbol{\omega}_{\mathrm{eff}}| = \sqrt{\left(\frac{2\mu E}{\hbar}\right)^2 + (\omega_0 - \omega)^2} \quad (15.1\text{-}28)$$

This motion is depicted in Figure 15.3, drawn for the initial condition $\mathbf{r}_{\mathrm{III}}(0) = \mathbf{a}_{\mathrm{III}}1$, which corresponds, according to (15.1-4), to an atom found initially in the upper state $|a\rangle$. The rotating I direction was chosen to coincide with the projection of ω on the 1–2 plane so that $\omega_{\mathrm{II}} = 0$.

Using basic trigonometric relations, we obtain from Figure 15.3,

$$\begin{aligned} r_{\mathrm{I}} &= \frac{\omega_{\mathrm{I}}(\omega - \omega_0)}{\omega_e^2}(1 - \cos \omega_e t) \\ r_{\mathrm{II}} &= -\frac{\omega_{\mathrm{I}}}{\omega_e} \sin \omega_e t \\ r_{\mathrm{III}} &= 1 - 2\left(\frac{\omega_{\mathrm{I}}}{\omega_e}\right)^2 \sin^2\left(\frac{\omega_e t}{2}\right) \end{aligned} \quad (15.1\text{-}29)$$

where $\omega_{\mathrm{I}} \equiv [-(2\mu E/\hbar)]$ is a negative number.

When we use (15.1-29) as well as the relation $r_{\mathrm{III}} = |a|^2 - |b|^2$ and the normalization condition, $|a|^2 + |b|^2 = 1$ results in

$$\begin{aligned} |a|^2 &= 1 - \left(\frac{\omega_{\mathrm{I}}}{\omega_e}\right)^2 \sin^2\left(\frac{\omega_e t}{2}\right) \\ |b|^2 &= \left(\frac{\omega_{\mathrm{I}}}{\omega_e}\right)^2 \sin^2\left(\frac{\omega_e t}{2}\right) \end{aligned} \quad (15.1\text{-}30)$$

for the probability of finding an atom, initially in the upper state, in states a (upper) and b, respectively.

At resonance ($\omega = \omega_0$), $\omega_e = \omega_{\mathrm{I}}$ and

$$\begin{aligned} |a|^2 &= \cos^2\left(\frac{\omega_{\mathrm{I}} t}{2}\right) \\ |b|^2 &= \sin^2\left(\frac{\omega_{\mathrm{I}} t}{2}\right) \end{aligned} \quad (15.1\text{-}31)$$

so that a complete population exchange between the upper and lower levels takes place every π/ω_{I} sec. Occupation probabilities $|a|^2$ and $|b|^2$ are plotted in Figure 15.4 for both the resonance and off-resonance conditions.

In closing, we should notice that in most experimental situations, an

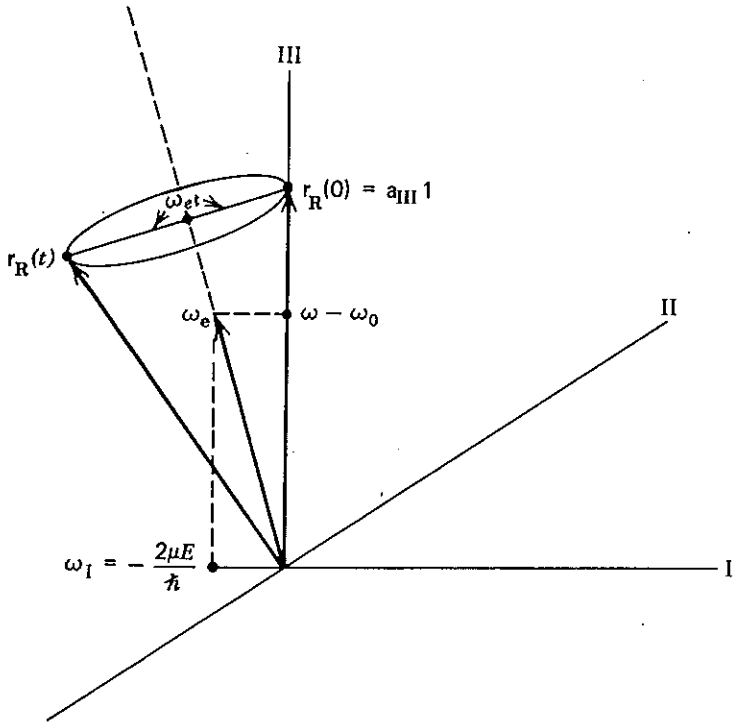

FIGURE 15.3 The motion of the vector $\mathbf{r}_R(t)$ in the rotating coordinate system (I, II, III). The motion consists of a precession at a rate $\omega_e = \sqrt{(\omega - \omega_0)^2 + (2\mu E/\hbar)^2}$ about the vector $\omega_{eff} = \mathbf{a}_I(-2\mu E/\hbar) + \mathbf{a}_{III}(\omega - \omega_0)$. The drawing corresponds to the initial condition $\mathbf{r}_R(0) = \mathbf{a}_{III}1$, that is, the atom is initially in the upper state $|a\rangle$.

atom is subjected to a linearly polarized field

$$E_x = E \cos \omega_0 t \tag{15.1-32}$$

rather than to the circularly polarized field (15.1-25) used in our formalism. The field of (15.1-32) can be resolved into two oppositely (circularly) polarized fields

$$\begin{aligned} E_{x1} &= \frac{E}{2} \cos \omega_0 t \\ E_{y1} &= \frac{E}{2} \sin \omega_0 t \end{aligned} \tag{15.1-33}$$

and

$$\begin{aligned} E_{x2} &= \frac{E}{2} \cos \omega_0 t \\ E_{y2} &= -\frac{E}{2} \sin \omega_0 t \end{aligned} \tag{15.1-34}$$

In a rotating coordinate system that is synchronous with the field (15.1-33), the field of (15.1-34) is seen to rotate at an angular rate of $2\omega_0$ so that to first order, it exerts no average "torque" on $\mathbf{r}$ and can be neglected. The resulting

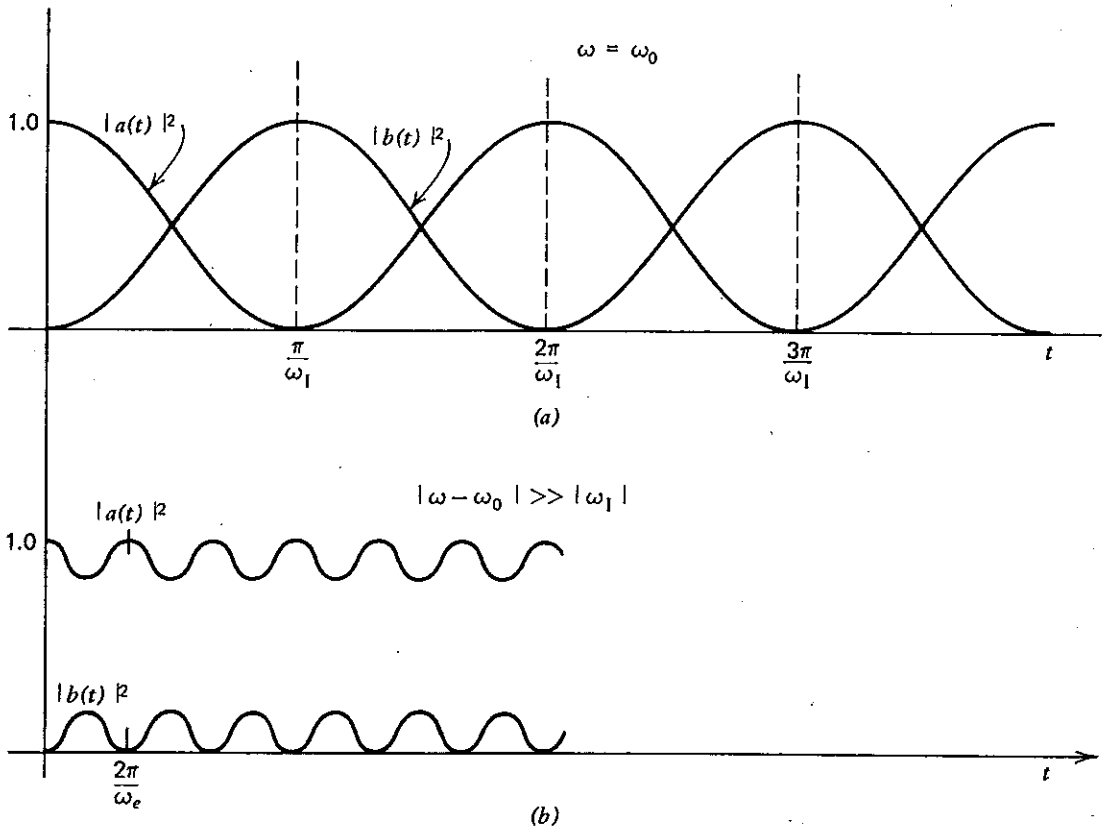

FIGURE 15.4 Oscillation of the occupation probability $|a|$ and $|b|$ when an optical field is applied. (*a*) $\omega = \omega_0$ ("on" resonance). (*b*) $|\omega - \omega_0| \gg |\omega_1|$.

motion of $\mathbf{r}$ is then the same as that studied above except that we must replace everywhere E by $E/2$.

15.2 SUPERRADIANCE

Consider the precession of the $\mathbf{r}$ vector in the (I, II, III) space under the influence of an applied circularly polarized field at a frequency $\omega_0 = \omega$ where $\hbar\omega$ is the transition energy. The atom is taken to be initially in the lower state $|b\rangle$, so that $\mathbf{r}_R(0) = -1\mathbf{a}_{III}$. Redrawing Figure 15.3 for this special case, we obtain the situation depicted in Figure 15.5. Let us turn the field off at a time t_0 where $|\omega_1|t_0 = \pi/2$. For $t \geq t_0$,

$$\begin{aligned} r_I &= r_{III} = 0 \\ r_{II} &= -1 \end{aligned} \tag{15.2-1}$$

and using (15.1-18) and (15.1-24), we obtain the oscillating atomic dipole moment in real space as

$$\begin{aligned} \mu_x &= \mu r_1 = \mu \sin \omega(t - t_0) \\ \mu_y &= \mu r_2 = -\mu \cos \omega(t - t_0) \end{aligned} \tag{15.2-2}$$

This is the largest dipole moment that can be "squeezed" out of one atom. If