Chem 249 Problem Set 2

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Three Phase Fresnel Calculation

1. Now that you've learned how to do it for two phases, create an Igor Pro file to calculate the Reflectivity for p-polarized and s-polarized light (R_p and R_s) as a function of incident angle from normal incidence (0 degrees) to grazing incidence (90 degrees) in 0.1 degree increments for the following THREE phase system where the index of refraction n_1 is real, and the indices of refraction of n_2 and n_3 can be complex:

i) n_1 := 1.51 (silica)
ii) n2 := 0.18 + 3.40i (gold)
iii) n3 := 1.00 + 0i (air)
iv) lambda = 633 nm
v) h := initial value 45.0, vary h in a box by increments of 1 nm.

See the Hansen Paper for detailed equations. For Igor Pro, a text file of the refined History emailed to me is the most useful, along with an initial print out.

Fluorescence Lifetime Measurements

2. Use LaPlace transforms to solve the following differential equation for fluorescence decay:

$$\frac{dC}{dt} = -\Gamma C$$

with the initial conditions C(t=0) = C_0.

3. Frequency domain fluorescence spectroscopy (fdfs) is an alternative method for obtaining fluorescence lifetimes. In these measurement, two parameters: a phase shift angle (phi) and demodulation factor (m), are measured. The mathematical relationship between these measured parameters and the fluorescence lifetime (tau) is stated (i) in the L. B. McGown paper on Modulation Fluorescence and (ii) the FLIM review, both available on the Website and in the Canvas Files/Handouts Folder). What are these relations, and from them can you deduce the transfer function H(s) for fluorescence modulation experiments?

Classical Simple Harmonic Oscillator and LaPlace Transforms

4. Consider the equation of motion for a damped harmonic oscillator:

$$m\frac{d^2x}{dt^2} + 2m\Gamma\frac{dx}{dt} + m\omega^2 x = 0$$

with the initial conditions x(0) = 0 and x'(0) = V_0. Use Laplace transforms to find x(t) for the two cases where $\omega >> \Gamma$ and $\omega = \Gamma$. Make plots in Igor of a couple of sample trajectories for these two cases.

Complex Frequency Dependent Susceptibility: Optical Absorption Spectrum

5. Consider the classical equation of motion for an electron in a damped harmonic well:

$$m\frac{d^2x}{dt^2} + 2m\Gamma\frac{dx}{dt} + m\omega_0{}^2 x = F(t)$$

where $F(t)$ is an externally applied Force. If $F(t)$ = qE(t) = -E_0 sin(ωt) , then the frequency dependent Polarization P(ω) = Ne x(ω) = χ(ω) E(ω) where N is the number of oscillators/volume, and χ(ω) is called the frequency dependent electric susceptibility

a) Show that χ(ω) has the form:

$$\chi(\omega) = \frac{S\omega_0^2}{\omega_0^2 - \omega^2 + i\omega\Gamma}$$

b) Please generate a plot of Imχ versus omega (a kind of Bode plot) for three cases where S = 0.1, ω_0 = 1000, and Γ = 10, 50, 100. Measure the FWHM (full width at half max) for the resonant peak. How does it scale with Γ?

c) Please generate a Nyquist plot (Imχ vs Reχ) for the case S = 0.1, ω_0 = 1000, and Γ = 50.

6. Using the general form for the general form for the electric susceptibility where S = 0.1, ω_0 = 1000, and Γ = 20, please plot the real and imaginary parts of n, the complex index of refraction (n = η + iκ), versus omega. Recall that $n^2 = \varepsilon_r = 1 + \chi$.