Photons and Electrons, Part 2



Erwin Schrödinger 1887-1961 Albert Einstein 1879-1955

The Photoelectric Effect (1905)



Photoelectric effect

$$h\nu = E_{kinetic} + \phi_{metal}$$

Wavelength dependence must be explained by the existence of quantized light.

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Problem: The work function of sodium metal is 2.280 eV, where $I = V = 1.602 \times 10^{-19}$ J. Find the wavelength of light capable of ejecting electrons from sodium metal with a speed of 4.000×10^{5} m s⁻¹.

Key equations: $hv = hc/\lambda = E_{kinetic} + \phi_{metal}$

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Step I: Calculate *E*_{kinetic} in eV.

 $E_{kinetic} = \frac{1}{2}mv^2 = (0.5)(9.109 \times 10^{-31} \text{ kg})(4.000 \times 10^5 \text{ m s}^{-1})^2$

 $E_{kinetic} = 7.288 \times 10^{-20} \text{ kg m}^2 \text{ s}^{-1} = 7.288 \times 10^{-20} \text{ Joules}$

 $E_{kinetic} = (7.288 \times 10^{-20} \text{ J})/(1.602 \times 10^{-19} \text{ J eV}^{-1})$

 $E_{kinetic} = 0.4549 \text{ eV}$

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Step 2: Calculate hv in eV.

hv = 0.4549 + 2.280 = 2.735 eV

note that we only keep 4 sig figs! We DO NOT report 2.7349 eV. **Problem:** The work function of sodium metal is 2.280 eV, where $I = V = 1.602 \times 10^{-19} J$. Find the wavelength of light capable of ejecting electrons from sodium metal with a speed of $4.000 \times 10^{5} m s^{-1}$.

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Step 3: Calculate λ in nm.

 $hv = 2.735 \text{ eV} = 1240./\lambda$

 $\lambda = 1240./2.735 = 453.4$ nm

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A note on significant figures:

4 sig figs: v = 4.000 x 10⁵ m s⁻¹ ; λ = 453.4 nm 3 sig figs: v = 4.00 x 10⁵ m s⁻¹ ; λ = 453 nm 2 sig figs: v = 4.0 x 10⁵ m s⁻¹ ; λ = 450 nm

Quantum Mechanics: The Schrödinger Equation



 $\hat{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$

Erwin Schrödinger 1887-1961



PIAB Energies and Wavefunctions

Energy eigenstates:

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Energy eigenstate wavefunctions:

$$\Psi_n(x) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right)$$
$$n = 1, 2, 3...$$



classical mechanics says the following:

- I. This particle can have **any energy**.
- 2. It can be located **anywhere**, with equal probability.



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- 3. At any instant in time, this particle will have a particular position that can be specified with arbitrarily high precision.



quantum mechanics makes a completely different set of predictions:

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2. We cannot know the exact trajectory and location of the particle in a given energy eigenstate. Only probabilities.

The wavefunctions for PIAB.

$$\Psi_n(x) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right)$$
$$n = 1, 2, 3 \dots$$

Note that $\Psi(0)=0$ and $\Psi(L)=0$. This is a property of standing waves in general (e.g., guitar or violin strings,).



What IS $\psi(\mathbf{x})$?

The one dimensional eigenstate wavefunction has no direct physical significance itself.

However, $\Psi^2(\mathbf{x})$ is the probability of finding our particle in the interval from x to x + dx.

"Born interpretation" (From Max Born, 1926).

$\Psi^2(\mathbf{x})$ is called the "probability density."

example: the probability of finding the particle in the region, say, from 0 to L (inside the box) is:

$$\left\langle \Psi_n \left\| \Psi_n \right\rangle = \int_{-\infty}^{+\infty} \psi_n^2(x) dx = \frac{2}{L} \int_{0}^{L} \sin^2 \left(\frac{n\pi x}{L} \right) dx = 1$$

for all n = 1, 2, 3...

$\Psi^2(\mathbf{x})$ is the probability density...

...here is what $\Psi^2(x)$ looks like for n=2 (left) and n=1 (right)...



e.g., for n=1, the particle "prefers" the center of the box!



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quantum mechanics makes a completely different set of predictions:

I. The particle can only have certain"allowed" values of energy.

2. We cannot know the exact trajectory and location of the particle in a given energy eigenstate. Only probabilities.

3. There is an inherent uncertainty in the position and momentum of the particle:

$$\Delta p \Delta x \ge \frac{\hbar}{2}$$

Heisenberg uncertainty principle



Heisenberg uncertainty principle

There are many Heisenberg pairs of observables. The two most important are:

 $\Delta p \Delta x \ge \frac{\hbar}{2}$ $\Delta E \Delta t \ge \frac{\hbar}{2}$







Some easier nomenclature:



States are sometimes just labelled with their quantum numbers

 $E_n = \frac{n^2 h^2}{8mL^2}$

Absorption and Emission Spectroscopy

 $|n = 1\rangle$



$$\Delta E_{51} = E_5 - E_1 = h\nu$$

Light will be absorbed or emitted only if $\Delta E = hv$

The spectrum is QUANTIZED!

 $\Delta E_{51}=24G$ $\Delta E_{41} = 15G$ $\Delta E_{31}=8G$ $\Delta E_{21}=3G$

 $G=h^2/8mL^2$



Atomic Emission Spectroscopy



The emission spectra of H, He and Hg are all quantized!

Atomic Emission Spectroscopy

Therefore, AES tells us that the electron energy levels in atoms are quantized.



Next up:

the quantum mechanical calculation of the electron energy levels in a Hydrogen atom.

Grotrian diagram for Hydrogen

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The photoelectric effect converts light into electrons. These electrons can be measured as a photocurrent.

Using the power supply to stop the photocurrent is a way of measuring the maximum electron K.E.

Photomultiplier tubes (PMTs) amplify the photocurrent from a alkali metal photocathode, creating pulses of a million electrons that can be detected.

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Discrete Output Pulses (Single Photon Event)

TIME

The photoelectron pulses from the PMT can be counted, yielding a measurement of photons per second. This detection method is called "photon counting".

If the number of photoelectron pulses from the PMT becomes too large to detect individually, an average photocurrent can be measured. Photon flux can be calculated from the Radiant Sensitivity (A/W) of the PMT.